Discrete Mathematics 21-228
Exam #4
Tuesday, June 22

You may use your notes and textbook. Show your work, and justify your results.

1. A *vertex cover* of a graph $G = (V, E)$ is a set $S \subseteq V$ that contains at least one endpoint of every edge.

   (a) What is the minimum size of a vertex cover for the following graphs: $P_n, C_n, K_n, K_{m,n}$?

   (b) Prove that if S is any vertex cover of G, and M is any matching of G, then $|S| \geq |M|$.

2. Let $T = (V, E)$ be a labeled tree on 8 vertices.

   (a) How many such trees are there?

   (b) Prove that the degree of a vertex $v \in V$ is one greater than the number of times v's label appears in the Prufer code for T.

   (c) Given the Prufer code for T, explain how you could tell whether T is a path.

   (d) Given the Prufer code for T, explain how you could tell whether T is a star.

   (e) How many labeled paths on 8 vertices are there?

   (f) How may labeled stars on 8 vertices are there?

3. **Defintion:** A *complete k-partite graph* $K_{n_1,n_2,\ldots,n_k}$ has vertices partitioned into k subsets of size $n_1, n_2, \ldots, n_k$ elements each, with vertices adjacent if and only if they are in different subsets of the partition.

   (a) For what values of n_1, n_2 does K_{n_1,n_2} have an Euler Tour?

   (b) Draw $K_{1,2,3}$ and $K_{2,2,2}$.

   (c) What is $\chi(K_{n_1,n_2,\ldots,n_k})$? Given a set of colors, how many different colorings are there?

   (d) For what values of $n_1, n_2, \ldots, n_k$ does $K_{n_1,n_2,\ldots,n_k}$ have an Euler Tour?

   (e) For what values of $n_1, n_2, \ldots, n_k$ does $K_{n_1,n_2,\ldots,n_k}$ have a Hamilton Cycle?