Discrete Mathematics 21-228
Exam #3
Friday, June 11

You may use your notes and textbook. Show your work, and justify your results.

Three identical-looking urns contain colored balls as follows: Urn 1 contains 1 red ball, 1 blue ball, and 1 green ball. Urn 2 contains 8 red balls, 2 blue balls and 2 green balls, and Urn 3 contains no red balls, 6 blue balls, and 3 green balls.

1. An urn is chosen at random, and then a ball is chosen at random from that urn.

   (a) Find the probability of drawing a green ball.

   (b) Are the events {Urn 1 is chosen} and {red ball is selected} independent? Prove that your answer is true.

   (c) What is the probability that a green ball is chosen given that Urn 2 is selected?

   (d) Given that a blue ball was selected, what is the probability that it came out of Urn 1?

2. Assume the procedure from problem 1 is done 75 times, with replacement– i.e. an urn is chosen, and a ball selected, its color is noted, and then the ball is returned to the urn where it came from. Then the process is repeated until 75 balls have been drawn.

   (a) What is the probability that the first red ball will be drawn on the fifth draw?

   (b) What is the probability that exactly 25 balls of each color will be drawn?

   (c) How many times do we expect to draw a red ball?

   (d) How many times do we expect to draw a ball which is the same color as the one drawn before it?