Discrete Mathematics 21-228
Exam #1
Friday, May 21

You may use your notes and textbook. Show your work, and justify your results.

1. Twenty-four indistinguishable balls are placed in six boxes.
   (a) In how many different ways can this be done?
   (b) In how many ways can this be done if each box must receive at least two balls?

2. Twenty-four distinguishable balls are to be placed in six boxes.

   (a) In how many ways can this be done?

   (b) In how many ways can this be done if box 1 must get 12 balls, box 2 must get 7 balls, box 3 must get 5 balls, and boxes 4, 5, and 6 must be empty?

3. Prove

$$\binom{n+1}{k} = \frac{n+1}{k}\binom{n}{k-1}$$

4. Prove

$$\sum_{k=0}^{m} \binom{m}{k}\binom{n}{r+k} = \binom{m+n}{m+r}$$

5. Extra Credit: Prove:

$$\binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + \cdots + n\binom{n}{n} = n2^{n-1}$$

More extra credit: provide more proofs of this identity.