Discrete Mathematics 21-228
Assignment #1 Solutions

2.
   - $A \cap B = \{x \in \mathbf{Z} : -5 \leq x \leq 16\}$
   - $B \cup C = \mathbf{Z}$
   - $(A \cup C) \cap B = \{x \in \mathbf{Z} : -5 \leq x \leq 16\} \cup \{x \in \mathbf{Z} : 17 < x$, and x is a multiple of $2\}$

3. In class we proved that the number of ways to color m indistinguishable balls with n colors is $\binom{m+n-1}{n-1}$. Imagine that each possible outcome of a die corresponds to coloring the die one of 6 colors. Then the number of ways to do this to n indistinguishable dice is $\binom{m+6-1}{6-1} = \binom{m+5}{5}$. As a reality check, note that when $m = 2$, we get $\binom{7}{5} = 21$ outcomes, as expected.

4. Again, we use the fact that the number of ways to color m indistinguishable balls with n colors is $\binom{m+n-1}{n-1}$. In this case, the dollar bills correspond to the indistinguishable objects, and the people correspond to colors: Think of giving a person a particular dollar bill as coloring the bill with that person's color. Thus with $m = 100$ and $n = 30$, there are $\binom{100+30-1}{30-1} = \binom{129}{29}$ ways to distribute the money.

   If each person must receive at least one dollar, then imagine giving each person a dollar at the beginning. After that, you are free to distribute the remaining \$70 in any way you see fit. There are $\binom{70+30-1}{30-1} = \binom{99}{29}$ ways to do this, using the same logic as above.