Finite Element Interpolation on Triangular Meshes

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An important part of the theory of the finite element method (for approximating solutions to partial differential equations) involves approximation of Sobolev functions by functions in finite dimensional subspaces. One such interpolation question is given below.

Question: Let $\mathcal{T}$ be a triangulation of a set $\Omega \in \mathbb{R}^2$. Given a function $u : \Omega \to \mathbb{R}$, can we find a function $u_{\mathcal{T}}$ which is continuous on Ω, affine on each triangle $t \in \mathcal{T}$, and a "good" approximation of u?

The answer to the question depends on three things: the smoothness of the function u, the triangles in $\mathcal{T}$, and the norm in which we are measuring the interpolation error. In this talk, we will discuss how the interpolation error depends on each of these things and generalize these error bounds to the case of piecewise polynomial interpolants.