The coarsening dynamics of faceted crystal surfaces

Experiments on the coarsening properties of a faceted crystal surface reveal a marked transition in behaviour between *annealing* and net growth regimes. We present a theory which explains this transition, and exhibit the main ideas in the setting where *attachment kinetics* is the dominant mass transfer mechanism. Here, the *Cahn-Hilliard* equation ($\mathcal{CCH}$) is the annealing model, while the *convective Cahn-Hilliard* ($\mathcal{FECH}$) equation is the associated growth model. We identify the sharp-interface theory of $\mathcal{CCH}$ through a matched asymptotic analysis. The result is a novel coarsening dynamical system ($\mathcal{CDS}$) with the following properties:

- The characteristic length $\mathcal{L}(t)$ scales like $t^{1/2}$.
- Binary coalescence of *phase boundaries* is impossible
- Coarsening may only occur through a specific coalescence of three phase boundaries; the *kink-ternary.*

These properties stand in marked contrast with with the generic binary coalescence and logarithmically slow coarsening of the 1-d Cahn-Hilliard equation. We thereby capture the *annealing-to-growth* transition alluded to above.

Motivated by a linear stability analysis we also propose a length-scale-doubling coarsening ansatz for the $\mathcal{CDS}$ evolution, from which we identify the scaling constant C of the $t^{1/2}$ scaling regime; e.g., $\mathcal{L}(t) = Ct^{1/2}$. This work is, in part, joint with Felix Otto (Bonn) and Stephen H. Davis (Northwestern).