Abstracts - Main speakers

E. Acerbi,
University of Parma
From p-laplacian to $p(x)$-laplacian

We discuss various regularity results for equations or systems related to the $p(x)$-laplacian operator $-\mathrm{div}\ |Du|^{p(x)-2}Du$, including elliptic or parabolic systems of the types arising from the theory of electrorheological fluids, and equations of the type

$$-\mathrm{div}\ |Du|^{p(x)-2}Du = -\mathrm{div}\ |f|^{p(x)-2}f\ ,$$

where $|f|^{q(x)} \in L^1$ for some function $q > p$, thereby proving higher integrability of Du. This is joint work with G. Mingione.

Giovanni Alberti,
University of Pisa,
Ginzburg-Landau functionals and the Plateau problem. A variational approach.

I will describe a Γ-convergence result for functionals of Ginzburg-Landau type obtained in collaboration with S. Baldo and G. Orlandi. The main consequence is the following: the Jacobian determinants of minimizers with suitable Dirichlet conditions converge to minimal surfaces M with prescribed boundary, while the corresponding energy densities converge to the area of M.

K. Bhattacharya,
California Institute of Technology
Free boundary problems in heterogeneous bodies

We discuss the homogenization of some free boundary problems that are motivated by the propagation of phase boundaries in heterogeneous bodies. This is based on joint work with Bogdan Craciun.

Andrea Braides,
University of Rome “Tor Vergata”,

Multiple scale effects in the passage from discrete to continuum variational problems

The study of variational limits of discrete energies allows to produce multi-scale phenomena from seemingly inoffensive pair potentials. In this way, we may recover continuum energies characteristic of theories of fracture, softening, nonlinear elasticity, phase transitions, non-local damage, etc., as first or second-order limit effects of simple lattice theories as the lattice spacing tends to zero. Conversely, the introduction of a new ‘microscopic dimension’ leads to original interesting phenomena, that often require a continuum formulation different from the ‘classical’ ones.

Giuseppe Buttazzo,
University of Pisa

Optimal networks in mass transportation problems

Mass transportation problems deal with a metric space (X, d) and two probability measures f^+ and f^- on X. The cost functional

$$F(d) = \inf \left\{ \int_{X\times X} d(x,y)\, d\gamma(x,y) \ : \ \gamma \text{ has marginals } f^+ \text{ and } f^- \right\}$$

can then be defined, and its value depends on the distance d we considered on X.
We give a model for the description of an urban transportation network and we consider the related optimization problem which consists in finding the design of the network which has the best transportation performances.

B. Dacorogna,
University of Lausanne
Viscosity and almost everywhere solutions of first order partial differential equations

Consider the differential inclusion $Du(x)$ belongs to E. We exhibit an explicit solution that we call fundamental. It turns out to be also a viscosity solution, when properly defining this notion. Finally we consider a Dirichlet problem associated to the differential inclusion and we give an iterative procedure for finding a solution.

G. Dal Maso,
SISSA, Trieste
Existence results for variational models in fracture mechanics

The talk will present some recent existence results for the variational model of crack growth in brittle materials proposed by Francfort and Marigo. These results, obtained in collaboration with Francfort and Toader, deal with the n-dimensional case, with a quasiconvex bulk energy and with time dependent prescribed boundary deformations and time dependent applied loads.

Georg Dolzmann,
University of Maryland
Variational problems in the analysis of solid-solid phase transformations

Materials undergoing solid-solid phase transformations often have surprising mechanical properties as, for example, the shape memory effect. In this talk, we study a remarkable consequence of the abundance of geometric patterns formed by the martensitic phases in a shape memory material. In fact, the material displays theoretically a fluid-like behavior in the sense that the set of all approximately energy free affine deformations has an open interior with respect to the constraint of incompressibility. We also address regularity questions for generalized minimizers of non-convex variational problems describing these materials.

Gero Friesecke,
Warwick

Minimum energy configurations of large atomistic systems

We discuss some examples of variational problems for large particle systems arising in physics, including

- a classical mechanics model for electrons in a large atoms
- a two-dimensional non-smooth interatomic force model introduced by Radin to explain crystallization
- a two-dimensional mass-spring model which can be proved to exhibit crystallization (joint work with F.Theil, using a recent continuum rigidity result obtained jointly with R.D.James and S.Müller).

Adriana Garroni,
University of Rome "La Sapienza"

Gamma-limit of a variational model for dislocations

We will study the asymptotic behaviour in terms of Γ-convergence of a variational formulation for a phase field model for dislocations. The energy is given by two main terms: the Peierls energy obtained by an infinitely many wells potential and a long range interaction elastic energy modelled by a non local term of $H^{1/2}$ type. To model a forest hardening we add zero boundary condition on a perforated domain. We analyse the asymptotic behaviour for different scales. One of the main tools is the introduction of suitable notion of non local capacity and the corresponding cell problem formula.

R. James,
University of Minnesota, Minneapolis

The possible existence of some interesting hybrid materials nominally forbidden by basic physics

These speculations begin with the observation by various scientists, probing new phenomena through the use of first principles studies, that the simultaneous occurrence of certain properties is unlikely. A prototype example is the simultaneous occurrence in a single material of ferromagnetism and

ferroelectricity. These studies typically do not consider the possibility of a first order phase transformation (hence the word "nominally" in the title), but there is a lot of indirect evidence that, if the lattice parameters are allowed to change significantly, then one might have co-existence of such properties. This seems to extend to diverse sets of nominally incompatible properties. Thus, one could try the following: seek a reversible first order phase transformation, necessarily also involving a distortion, from, say, ferroelectric to ferromagnetic phases. If it were highly reversible, there would be the extremely interesting additional possibility of controlling the volume fraction of phases with fields or stress. The key point is reversibility.
Even big first order phase changes can be highly reversible (liquid water to ice, shape memory materials), and we argue that in solid materials it is the nature of the shape change that is critical. We suggest, based on a close examination of measured hysteresis loops in martensitic systems, that an idea based on "good fitting of the phases" governs reversibility. The mathematical status of this idea is not completely clear, as I explain, but its possible connection with quasiconvexity raises mathematical questions of fundamental interest.

Robert V. Kohn,
Courant Institute, New York

A continuum approach to structure of twist grain boundaries

The structure of a twist-grain boundary is a lattice of screw dislocations separating a periodic array of regions with good registry. This structure is usually explained using atomic-scale models. My talk presents recent joint work with Antti Pihlaja (Merrill Lynch) exploring a different, continuum approach. We view the pattern as arising from minimization of an "incoherence energy" $\int W(H)\,dx_1\,dx_2$ where H is the incoherency tensor of Cermelli and Gurtin. The rows of H are curl-free, so this resembles a problem of 2D elasticity. In particular, when W is not quasiconvex, minimizing sequences develop spatial patterns. Our main contributions are (a) identification of the patterns and Young measures associated with the standard picture of a twist grain boundary; and (b) identification of a specific energy density W for which this pattern is optimal. The identification of a physically plausible W – representing, perhaps, the continuum limit of an atomic-scale model – remains an open problem.

Jan Malý,
Charles University, Prague

Scalar minimizers with fractal singular sets

Lack of regularity of local minimizers for convex functionals with non-standard growth conditions is considered. It is shown that there exists a function $a \in C^{\alpha}(\Omega)$ such that the functional

$$\mathcal{F} : u \mapsto \int_{\Omega} (|Du|^p + a(x)|Du|^q)\, dx$$

admits a local minimizer $u \in W^{1,p}(\Omega)$ whose set of non-Lebesque points is a closed set Σ with Hausdorff dimension arbitrarily close to $N - p$. Here $\Omega \subset \mathbb{R}^N$ is an open set and $1 < p < N < N + \alpha < q < +\infty$. This is joint work with Irene Fonseca and Giuseppe Mingione.

Paolo Marcellini,
University of Florence

Implicit partial differential equations

With the title of *Implicit partial differential equations*, we refer to differential equations or systems of **first** order (we consider systems of second order too) of the form

$$\begin{cases} F_1(x, u(x), Du(x)) = 0 \\ F_2(x, u(x), Du(x)) = 0 \\ \qquad \cdots \\ F_I(x, u(x), Du(x)) = 0 \end{cases}, \qquad \text{a.e. } x \in \Omega,$$

where $\Omega \subset \mathbb{R}^n$ is an open set, $u : \Omega \subset \mathbb{R}^n \to \mathbb{R}^m$ and therefore $Du \in \mathbb{R}^{m \times n}$ (if $m = 1$ we say that the problem is *scalar* and otherwise we say that it is *vectorial*), $F_i : \Omega \times \mathbb{R}^m \times \mathbb{R}^{m \times n} \to \mathbb{R}$, $F_i = F_i(x, s, \xi)$, $i = 1, \ldots, I$, are given. The equations that we will consider are called of *implicit type*, since they exclude the quasilinear case, i.e., the case when the functions F_i are linear with respect to the gradient Du.
We present some work in collaboration with Bernard Dacorogna; some references are:

B.Dacorogna and P.Marcellini, *General existence theorems for Hamilton-Jacobi equations in the scalar and vectorial case*, Acta Mathematica, 178 (1997), 1–37.

B.Dacorogna and P.Marcellini, *Implicit partial differential equations*, Progress in Nonlinear Differential Equations and their Applications, 37, Birkhäuser, Boston, 1999.

and more recently:

B.Dacorogna and P.Marcellini, *Viscosity solutions, almost everywhere solutions and explicit formulas*, December 2002, preprint.

Abstracts - Short Communications

Y. Altundas,
University of Minnesota

Parallel Computation of Phase Field Model and Comparision with the Experiment

In this talk, we will present the results obtained from 3D calculation of a phase field model along with a movie of dendritic growth of a single needle crystal. The properties of dendritic growth such as interface growth velocity and the influence of anisotropy on the dendritic growth will be compared with the microgravity experiment. tip radius of the dendrite will be compared
The results will be compared with microgravity experiments for Succinonitrile.

M. Camar-Eddine,
University of Utah

Determination of the closure of the set of elasticity functionals

We determine the closure for the Mosco-convergence in $L^2(\Omega, \mathbb{R}^3)$ of the set of elasticity functionals. We prove that this closure coincides with the set of all non-negative lower-semicontinuous quadratic functionals which are objective, i.e. which vanish for rigid motions. The result is still valid if we consider only the set of isotropic elasticity functionals which have a prescribed Poisson coefficient. This shows that a very large family of materials can be reached when homogenizing a composite material with highly contrasted rigidity coefficients.

A. Domokos,
University of Pittsburgh

$C^{1,\alpha}$-regularity for the p-Laplacian in the Heisenberg group

Let Ω be a bounded domain in the Heisenberg group $\mathbb{H}^n$ and let $p > 1$. Consider the p-Laplace equation:

$$\sum_{i=1}^{2n} X_i \left(|Xu|^{p-2} X_i u \right) = 0 \,, \quad \text{in } \Omega \tag{1}$$

where $Xu = (X_1 u, ..., X_{2n} u)$ is the horizontal gradient of u and for $1 \leq i \leq n$

$$X_i = \frac{\partial}{\partial x_i} - \frac{x_{n+i}}{2} \frac{\partial}{\partial t} \,,$$

$$X_{n+i} = \frac{\partial}{\partial x_{n+i}} + \frac{x_i}{2} \frac{\partial}{\partial t}$$

are the left invariant vector fields corresponding to the canonical basis of the Lie algebra of the Heisenberg group. Let us denote by

$$T = \frac{\partial}{\partial t} \,.$$

We will discuss some recent results regarding the regularity of the weak solutions of equation (1), improving related results of Hörmander, Capogna and Marchi.

Our main contribution is an iteration scheme in the T-direction. Along the way we prove a result on fractional difference quotients that allows a more direct proof than the use of Besov spaces and opens the way to further improvements of our results.

L. Freddi,
University of Udine
A Young measure theory for martensitic thin films via dimension reduction

A variational limit defined on the space of bi-dimensional gradient Young measures is obtained from three-dimensional elasticity via dimension reduction. The obtained limit problem uniquely determines the energy density of the thin film. Our result might be used to compute the microstructure in membranes made of phase transforming material.
Preprint available at the web address http://cvgmt.sns.it/papers/frepar03

N. Jung,
University of Illinois at Urbana-Champaign
Continuity Properties of Distributional Determinant of the Hessian

Motivated by the statisticians' problem of minimizing a functional involving $\int |\det D^2u|$ in higher dimensional spaces, I define the distributional determinant of the Hessian on $BV^2 \cap W^{1,\infty}(\mathbb{R}^3)$ which includes possible minimizers of the functional, that is, piecewise linear functions, and prove its continuity by introducing the notion of strict convergence and minimal liftings.

V. Kucher,
University of San Diego
The second variation of a two-phase potential and some necessary conditions for local minimizers at an interface

This study is motivated by a problem of classification of coexisting equilibrium deformations in nonlinear elasticity. A variational problem for the functional with two-phase energy density is considered. It is well-known that if a piecewise smooth local minimizer exists then, from the vanishing-first variation, the minimizer satisfies the Weierstrass-Erdmann conditions at the surface of discontinuty of the gradient. A natural question is whether the second order necessary conditions at the interface exist. To take into account a variation of the interface both dependent and independent variables

are altered. The corresponding formula for the second variation of the functional is obtained. The localization theorem leads to the integral inequality at internal points of the interface. Some consequences of this inequality are discussed. In particular, connection between the Weierstrass-Erdmann conditions and necessary conditions for the positivity of quadratic functionals obtained by H.Simpson and S.Spector is shown. I am very grateful to Professor A.Freidin and Professor V.Osmolovskii for the advice and numerous discussions.

S. Levine,

Duquesne University, Pittsburgh

Nonstandard Growth Functionals in Image Restoration

We present a functional of $p(x)$ growth ($p(x) \geq 1$) for which the corresponding minimization problem provides a model for image restoration. The model results in total variation based diffusion along object boundaries and isotropic diffusion in homogeneous regions. At all other locations, the type of anisotropy varies according to the local image information. Existence, uniqueness, and asymptotic behavior of the minimization problem and it's related flow are established.Experimental results show the effectiveness of the model in image restoration.

H. Merdan,

University of Pittsburgh

Renormalization group methods and regimes in interface problems

In this talk, we will discuss the temporal evolution of an interface separating two phases for its large time behavior. The calculation involves adaptation of methodology known in the physics literature as renormalization group and scaling theory.

We will consider a full two-phased model in the quasi-static regime. Implementing a renormalization procedure we will calculate the characteristic length, $R(t)$, of a self-similar system that is the time dependent length scale characterizing the pattern growth. When the dynamical undercooling is non-zero ($\alpha \neq 0$), we will show that $R(t)$ increases as $t^{-1/\lambda}$ while the total

surface area of the interface, $S(t)$, increases as $t^{(1-d)/\lambda}$, where λ can take on values in the continuous spectrum, $[-3,-2]$. For $\alpha = 0$ the spectrum is $[-3,0)$ so that the single value of $\lambda = -1$ is selected by the plane wave imposed by Jasnow and Vinals. It will be also shown that the capillarity length arising from surface tension is irrelevant for the large time behavior even though it has a crucial role at the early stage evolution of an interface. At the end, the results will be compared with those of Jasnow and Vinals, and Caginalp.

J. S. Moll,
University of Valencia

The Total Variation Flow with Measure Initial Conditions

We consider the minimizing total variation flow in $\mathbb{R}^N$

$$\frac{\partial u}{\partial t} = \operatorname{div}\left(\frac{Du}{|Du|}\right) \qquad \text{in } Q_T =]0,T[\times\mathbb{R}^N, \tag{2}$$

coupled with the initial condition

$$u(0) = \mu, \qquad \mu \text{ being a Radon measure in } \mathbb{R}^N. \tag{3}$$

This PDE appears (in a bounded domain Ω) in the steepest descent method for minimizing the total variation, a method introduced by L. Rudin, S. Osher and E. Fatemi in the context of image denoising and reconstruction.

In previous papers thes gradient descent flow (2) has been studied in a bounded domain under Neumann and Dirichlet boundary conditions, obtaining existence and uniqueness results for initial data in $L^1(\Omega)$. Our aim is to continue the study of the flow (2) when the initial conditions are Radon measures in $\mathbb{R}^N$. We shall not consider general measures, instead we shall restrict ourselves to the case of measures

$$\mu = h + \alpha\mathcal{H}^k \llcorner S, \tag{4}$$

where $h \in L^1(\mathbb{R}^N) \cap L^\infty(\mathbb{R}^N)$, $\alpha \geq 0$, $\mathcal{H}^k$ is the k-dimensional Hausdorff measure in $\mathbb{R}^N$ and S is a k-manifold in $\mathbb{R}^N$ of class $W^{3,\infty}$.

We study limit solutions obtained by weakly approximating the initial measure μ by functions in $L^1(\mathbb{R}^N)$, characterizing these limit solutions when the initial condition $\mu = h+\mu_s$ where $h \in L^1(\mathbb{R}^N)\cap L^\infty(\mathbb{R}^N)$, and $\mu_s = \alpha\mathcal{H}^k\llcorner S$,

$\alpha \geq 0$, k is an integer and S is a k-dimensional manifold with bounded principal curvatures. In case $k < N-1$ we prove that the singular part of the solution does not move, it remains equal to μ_s for all $t \geq 0$ and its absolutely continuous part is the unique strong solution of the Cauchy problem associated with (2) with initial datum h. In particular, $u(t) = \delta_0$ when $u(0) = \delta_0$. In case $k = N-1$ we prove that the singular part of the limit solution is $(1 - \frac{2}{\alpha}t)^+ \mu_s$ and we also characterize its absolutely continuous part.
This is joint work with F. Andreu (University of Valencia), V. Caselles (University of Pompeu-Fabra) and J. M. Mazón (University of Valencia).

I. Nikolova,
University of Pittsburgh

Level Set-like Method Applied to Solving Markstein (Eiconal) Equation

Phenomena of moving surfaces arises in many areas, such as fluid mechanics, combustion, material science [1-3]. Recently, a paper on methods to study interfaces, [1] drew our attention. Among the methods describing moving surfaces are different geometry methods in combination with functional analysis approach, which is the one adopted in [2]. The method treats the flame as a surface of discontinuity, given by the function $(x, y, t) = const.$ The surface propagates (advances) in a normal direction with a speed F. The formulation of the idea, given in [1] is very close, if not the same as the one given in [2], but was published 8 years later. In this paper we derive the eiconal equation, which is known as Markstein equation in combustion community. This equation describes the flame front propagation, $= const.$ Equation for the evolution of is solved by upwind finite differences method. In modern literature [3, 4] the method is referred to as "level set method". The front position at the time t is given by the zero level set of evolving function. Results on the flame front position in vortex structured flow fields are presented illustrating the features of the method applied to this problem. Depending on the type of the flow field, possibility for the flame to be "trapped" arises.

References:

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3. Osher, S, and Sethian, J.A., J. Computational Physisc, 79, 12 (1988).

4. Sethian, J.A., Level set methods, Cambridge University Press, UK, 1999

Acknowledgements: This study was inspired by my visit to the University of Bristol in the frame of the EC program “Go West”. For the references and discussions on the topic I would like to thank my former colleague and friend Sonia Tabakova from the Technical University of Plovdiv.

G. Pisante,
University of Lausanne

Existence of viscosity solution – Sufficient conditions for the existence of viscosity solutions for non convex Hamiltonians

We study the Dirichlet problem for the Hamilton-Jacobi equation

$$\begin{cases} F(Du) & = & 0 & \text{in} & \Omega \\ u & = & \varphi & \text{on} & \partial\Omega \end{cases} \tag{5}$$

where $\Omega \subset \mathbb{R}^n$ is a bounded open set, $F : \mathbb{R}^n \to \mathbb{R}$ is continuous and $\varphi : \partial\Omega \to \mathbb{R}$ is Lipschitz.
Under some technical assumptions on F, we give a sufficient geometric condition for the existence of Lipschitz viscosity solutions of problem (5). In particular we show an Hopf-Lax type extension formula in the case where we drop the classical hypotheses of convexity and regularity on F and Ω.

C. Rios,
Mc Master University

A higher dimensional partial Legendre transform, and regularity of degenerate Monge-Ampére equations

In dimension $n \geq 3$, we define a generalization of the classical two dimensional partial Legendre transform, that reduces interior regularity of the generalized Monge-Ampére equation $\det D^2u = k\,(x, u, Du)$ to regularity of a divergence form quasilinear system of special form. This is then used to obtain smoothness of $C^{2,1}$ solutions, having $n-1$ nonvanishing principal curvatures, to certain subelliptic Monge-Ampére equations in dimension $n \geq 3$. A corollary is that if $k \geq 0$ vanishes only at nondegenerate critical points,

then a $C^{2,1}$ convex solution u is smooth if and only if the symmetric function of degree $n-1$ of the principal curvatures of u is positive, and moreover, u fails to be $C^{3,1-\frac{n}{2}+\varepsilon}$ when not smooth.
This is joint work with Eric Sawyer, McMaster University, and Richard Wheeden, Rutgers University.

A. Tudorascu, Carnegie Mellon University General Fokker-Planck Equations via Monge-Kantorovich-Theory

The generic Fokker-Planck equation describes the evolution of the probability density for a stochastic process associated with an Ito stochastic differential equation. Here we employ elements of the Monge-Kantorovich mass transfer theory to study general (possibly degenerate) Fokker-Planck equations.

C. Wang, University of Kentucky The Aronsson equation for absolute minimizer of L-infinity functional associated with vector fields satisfying Hormander's condition

In this talk, I will discuss the Aronsson equation for absolute minimizer, i.e. local minimizers, of certain L^∞ functionals with respect to the CC metric space generated by Hormander's vector fields and its Euler-Aronsson equation and its uniqueness when the CC metric is induced from a Carnot group.

S. Watson, Northwestern University, Chicago Coarsening dynamics of faceted crystal surfaces

The coarsening properties of a faceted crystal surface *annealing* (in equilibrium) with it's melt is known experimentally to change dramatically if, instead, the crystal is subject to net growth. We present a theory which explains this transition, and exhibit the main ideas in the setting where *attachment kinetics* is the dominant mass transfer mechanism. Here, the *faceting-Cahn-Hilliard* ($\mathcal{FCH}$) equation is the annealing model, while the *faceting-eikonal-Cahn-Hilliard* ($\mathcal{FECH}$) equation is the associated growth model. We identify the sharp-interface theory of $\mathcal{FECH}$ through a matched asymptotic analysis. The result is a novel coarsening dynamical system ($\mathcal{CDS}$) for the edge network of the faceted surface, where coarsening occurs through the merging and annihilation of facets. In the case of square symmetry, we prove for $\mathcal{CDS}$ that

- The characteristic length $\mathcal{L}(t)$ of the faceted surface scales like $t^{1/2}$.
- *Pyramids* are stable and *anti-pyramids* are unstable.

Direct numerical simulations of $\mathcal{FECH}$ also yield this scaling law and confirm the stability-instability dichotomy through the absence of anti-pyramids in the surface evolution. Hence, our results capture the *annealing to growth transition* of the coarsening dynamics since, for $\mathcal{FCH}$, $\mathcal{L}(t) \sim t^{1/4}$ and also both pyramids and anti-pyramids appear in the evolution.
The one-dimensional $\mathcal{FECH}$, which is equivalent to the convective Cahn-Hilliard $\mathcal{CCH}$ equation, displays unusual coarsening mechanisms. We prove that

- Binary coalescence of *phase boundaries* is impossible
- Coarsening may only occur through a specific coalescence of three phase boundaries; the *kink-ternary.*

These properties stand in marked contrast with with the generic binary coalescence of the 1-d Cahn-Hilliard equation.
Motivated by a linear stability analysis we also propose a length-scale-doubling coarsening ansatz for the $\mathcal{FECH}$ evolution, from which we identify the scaling constant C of the $t^{1/2}$ scaling regime; e.g., $\mathcal{L}(t) = Ct^{1/2}$. This work is, in part, joint with Felix Otto (Bonn)

Q. Xia,

University of Texas

Variational problems in the intersection homology theory of singular varities

In this talk, I am going to discuss "soap films" or "minimal surfaces" that live in singular spaces such as singular complex projective varieties. On complex projective varieties, the intersection homology theory introduced by MacPherson and Goresky is more suitable than ordinary homology theory. I will first discuss how to express the intersection homology groups in terms of integer multiplicity rectifiable currents. These are then isomorphic to the usual intersection homology groups defined by geometric or subanalytic chains with the corresponding perversity conditions. After that, I will discuss the existence and regularity of modified mass minimizer in every intersection homology class on a compact stratified subanalytic pseudomanifold. My modified mass minimizers may intersect the singular locus of the singular variety in a controlled fashion.

X. Yao,

Texas A&M Univ.

A Minimax Method for Finding the Eigenpairs of Nonlinear Operators

A minimax algorithm for finding the eigenpairs of nonlinear operators in Banach spaces will be given in this talk. As a preliminary result, the following type of variational eigenpair problems will be considered. Find eigenpairs $(\lambda, u) \in \mathcal{R} \times B$ such that $\|u\| = 1$ and

$$F'u = \lambda G'u$$

where B is a Banach space and $F, G \in C^1(B, R)$ satisfy the homogeneous conditions

$$F(tu) = t^k F(u) \quad \text{and} \quad G(tu) = t^k G(u) \quad \forall t > 0, \text{ for some } \ k \neq 0.$$

As examples, the eigenpairs of the nonlinear *p-Laplacian* and *pseudo-p-Laplacian* operators will be studied and their numerical results will be presented.
This is a joint work with Professor J. Zhou of Texas A&M University.

J. Zimmer,
California Institute of Technology

On the computation of nonconvex hulls

An open problem in the calculus of variations is the computation of the quasiconvex envelope of a given set of matrices. Also, main results in relaxation theory rely on the quasiconvex envelope of a function. Such a envelope or hull, however, is known only for very specific cases.
Therefore, we want to study two related, but simpler, notions of convexity, namely separate convexity and rank-1-convexity. We present a graph-theoretical method for the computation of the separate convex envelope of a finite set of matrices.
For rank-1-convexity, we want to answer the following question: Given a finite set of matrices, does it contain a Tartar square? Tartar squares are known to be a main obstruction for nontrivial rank-1-convex hulls, and indeed new results indicate that the algorithm presented in the talk indeed can show triviality of the rank-1-convex hull for a large class of configurations.
Unlike previous approaches, we do not build on numerical methods (in the sense of discretization). Rather, we use methods of computational algebraic geometry to avoid mistakes due to rounding errors and get exact results in a few seconds.