Paolo Marcellini

Implicit partial differential equations

With the title of *Implicit partial differential equations*, we refer to differential equations or systems of **first** order (we consider systems of second order too) of the form

$$\begin{cases} F_1(x,u(x),Du(x))=0 \\ F_2(x,u(x),Du(x))=0 \\ \cdots \\ F_I(x,u(x),Du(x))=0 \end{cases}, \qquad \text{a.e. } x\in\Omega, \tag{1}$$

where $\Omega\subset\mathbb{R}^n$ is an open set, $u:\Omega\subset\mathbb{R}^n\to\mathbb{R}^m$

and therefore $Du\in\mathbb{R}^{m\times n}$

(if $m=1$ we say that the problem is *scalar* and otherwise we say that it is *vectorial*),

$F_i:\Omega\times\mathbb{R}^m\times\mathbb{R}^{m\times n}\to\mathbb{R}$,

$F_i=F_i(x,s,\xi)$, $i=1,\dots,I$, are given.

The equations that we will consider are called of *implicit type*, since they exclude the quasilinear case, i.e., the case when the functions F_i are linear with respect to the gradient Du.

We present some work in collaboration with Bernard Dacorogna; some references are:

B.Dacorogna and P.Marcellini, *General existence theorems for Hamilton-Jacobi equations in the scalar and vectorial case*, Acta Mathematica, 178 (1997), 1–37.

B.Dacorogna and P.Marcellini, *Implicit partial differential equations* , Progress in Nonlinear Differential Equations and their Applications, 37, Birkhäuser, Boston, 1999.

and more recently:

B.Dacorogna and P.Marcellini, *Viscosity solutions, almost everywhere solutions and explicit formulas*, December 2002, preprint.