Scalar minimizers with fractal singular sets

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Lack of regularity of local minimizers for convex functionals with non-standard growth conditions is considered. It is shown that there exists a function $a \in C^\alpha(\Omega)$ such that the functional

$$\mathcal{F} : u \mapsto \int_\Omega (|Du|^p + a(x)|Du|^q)\, dx$$

admits a local minimizer $u \in W^{1,p}(\Omega)$ whose set of non-Lebesque points is a closed set Σ with Hausdorff dimension arbitrarily close to $N - p$. Here $\Omega \subset \mathbb{R}^N$ is an open set and $1 < p < N < N + \alpha < q < +\infty$.