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The second variation of a two-phase potential and some necessary conditions for local minimizers at an interface

This study is motivated by a problem of classification of coexisting equilibrium deformations in nonlinear elasticity. A variational problem for the functional with two-phase energy density is considered. It is well-known that if a piecewise smooth local minimizer exists then, from the vanishingfirst variation, the minimizer satisfies the Weierstrass-Erdmann conditions at the surface of discontinuty of the gradient. A natural question is whether the second order necessary conditions at the interface exist. To take into account a variation of the interface both dependent and independent variables are altered. The corresponding formula for the second variation of the functional is obtained. The localization theorem leads to the integral inequality at internal points of the interface. Some consequences of this inequality are discussed. In particular, connection between the Weierstrass-Erdmann conditions and necessary conditions for the positivity of quadratic functionals obtained by H.Simpson and S.Spector is shown. I am very grateful to Professor A.Freidin and Professor V.Osmolovskii for the advice and numerous discussions.