

**Department of Mathematical Sciences**  
**Carnegie Mellon University**  
21-393 Operations Research II  
Test 2

Name: \_\_\_\_\_

Problem	Points	Score
1	35	
2	35	
3	30	
Total	100	

**Q1: (35pts)**

Use the KKT conditions to solve

Minimise  $(x_1 - 2)^2 + (x_2 - 2)^2$  subject to  $x_1 + 2x_2 \leq 3, 2x_1 + x_2 \leq 1$ .

**Q2: (35pts)** There are two machines available for the processing of  $n$  jobs. The processing time of job  $j$  is  $p_j > 0$  for  $j = 1, 2, \dots, n$ . The objective is to assign jobs to machines in order to minimise  $C_{\max} = \max\{C_j : j = 1, 2, \dots, n\}$  where  $C_j$  is the completion time of job  $j$ . Let

$$P_1 = \sum_{j=1}^n p_j \text{ and } P_2 = \max\{p_j : j = 1, 2, \dots, n\}$$

Show that the optimal solution satisfies

$$\frac{P_1}{2} \leq C_{\max} \leq \frac{P_1 + P_2}{2}.$$

**Q3: (30pts)**

In an inventory system for a single product there is a cost of  $AQ^{1/2}$  for making an order of size  $Q$ . No stockouts are allowed. The inventory cost per period is  $Ih^{1/2}$  where  $h$  is the average amount of stock held. Determine an optimal purchasing/stock strategy.