Department of Mathematical Sciences
Carnegie Mellon University

21-393 Operations Research II
Test 2

Name:____________________________________

Problem	Points	Score
1	33	
2	33	
3	34	
Total	100	

Q1: (33pts)
Formulate the following as an integer program:
Suppose that a state sends R persons to the U.S. House of Representatives. There are $D > R$ counties in the state and the state legislature wants to group these counties into R distinct electoral dstricts, each of which sends a delegate to Congress. The total population of the state is P, and the legislature wants to form districts whose population approximates $p = P/R$. Suppose that the appropriate legislative committee studying the electoral districting problem generates a long list of $N > R$ *candidates* to be districts. Each of the candidates contains contiguous counties and the total population of candidate j is p_j, $j = 1, 2, \ldots, N$. Define $\pi_j = |p_j - p|$ and

$$a_{i,j} = \begin{cases} 1 & \text{if county } i \text{ is included in candidate } j \\ 0 & \text{otherwise} \end{cases}$$

Given the values of $p_j, a_{i,j}$, the objective is to select R of these candidates such that each county is contained in a single district and such $\max\{\pi_j\}$ is as small as possible.

Q2: (33pts)
Analyse the following inventory system and derive a strategy for minimising total cost. There are n products. Product i has demand λ_i per period and no stock-outs are allowed. The cost of making an order for Q units of a mixture of products is AQ^α. The inventory cost is $I \max\{L_1, L_2, \ldots, L_n\}$ per period where L_i is the inventory level of product i in that period.

Q3: (34pts) There are two machines available for the processing of $n = 2m$ jobs. The processing time of job j is $p_j > 0$ for $j = 1, 2, \dots, n$. The objective is to assign jobs to machines in order to minimise $\sum_{j=1}^{n} C_j$ where C_j is the completion time of job j. Let $m_i, i = 1, 2$ denote the number of jobs executed on machine i.

1. Suppose that machine 1 processes jobs $i_1, i_2, \dots, i_s$ and machine 2 processes jobs $j_1, j_2, \dots, j_t$ in this order. Show that the contribution of machine 1 to the objective function is

$$sp_{i_1} + (s-1)p_{i_2} + \cdots + 2p_{i_{s-1}} + p_{i_s}.$$

2. Show that in the optimal solution, each machine processes m jobs.

3. Show that $p_{i_1} \leq p_{i_2} \leq \cdots \leq p_{i_m}$.

4. Show that $p_{i_m} \geq p_{j_{m-1}}$.

Using 3. and 4. deduce the structure of an optimal solution.