Department of Mathematical Sciences
Carnegie Mellon University

21-393 Operations Research II
Test 2

Name:______________________________________

Problem	Points	Score
1	33	
2	33	
3	34	
Total	100	

Q1: (33pts)

Consider the Knapsack problem:

$$\begin{array}{lllllll} \text{Maximise} & p_1x_1 & + & p_2x_2 & + \cdots + & p_nx_n & \\ \text{Subject to} & & & & & & \\ & w_1x_1 & + & w_2x_2 & + \cdots + & w_nx_n & \leq \quad W \\ & \multicolumn{6}{l}{x_1, x_2, \ldots, x_n \geq 0 \text{ and integer.}} \end{array}$$

Assuming that $p_1/w_1 \geq p_2/w_2 \geq \cdots \geq p_n/w_n$ the **Greedy Algorithm** puts $x_1 = \lfloor W/w_1 \rfloor, x_2 = \lfloor (W - w_1x_1)/w_2 \rfloor, x_3 = \lfloor (W - w_1x_1 - w_2x_2)/w_3 \rfloor$ and so on. Show that the value of the solution produced is always at least half the value of the optimal solution.

Q2: (33pts) A scout is going on a trip. She must select from a set of n items. The items fall into m types and T_i is the set of items of type i. Item j is of value v_j for $j = 1, 2, \ldots, n$. There are some restrictions on what she can take:

1. She can take between a_i and b_i items of type i for $i = 1, 2, \ldots, m$.
2. There is a list of pairs of items L_1 such that if $(j, k) \in L_1$ then she cannot take both of items j and k.
3. There is a list of triples of items L_2 such that if $(i, j, k) \in L_2$ then if she takes both of items i and j, then she must also take item k.

Construct an integer program that will solve the problem of maximising the total value of the items she can take.

Q3: (34pts) Find the optimal ordering strategy for the following inventory system. If you order an amount Q, it arrives immediately and the cost of the order is A. The inventory cost is I per unit of inventory per unit of time. The demand per period, t units of time after an order is $2(Q^{1/2} - t)$ and no stock-outs are allowed. You must re-order when the inventory reaches zero.