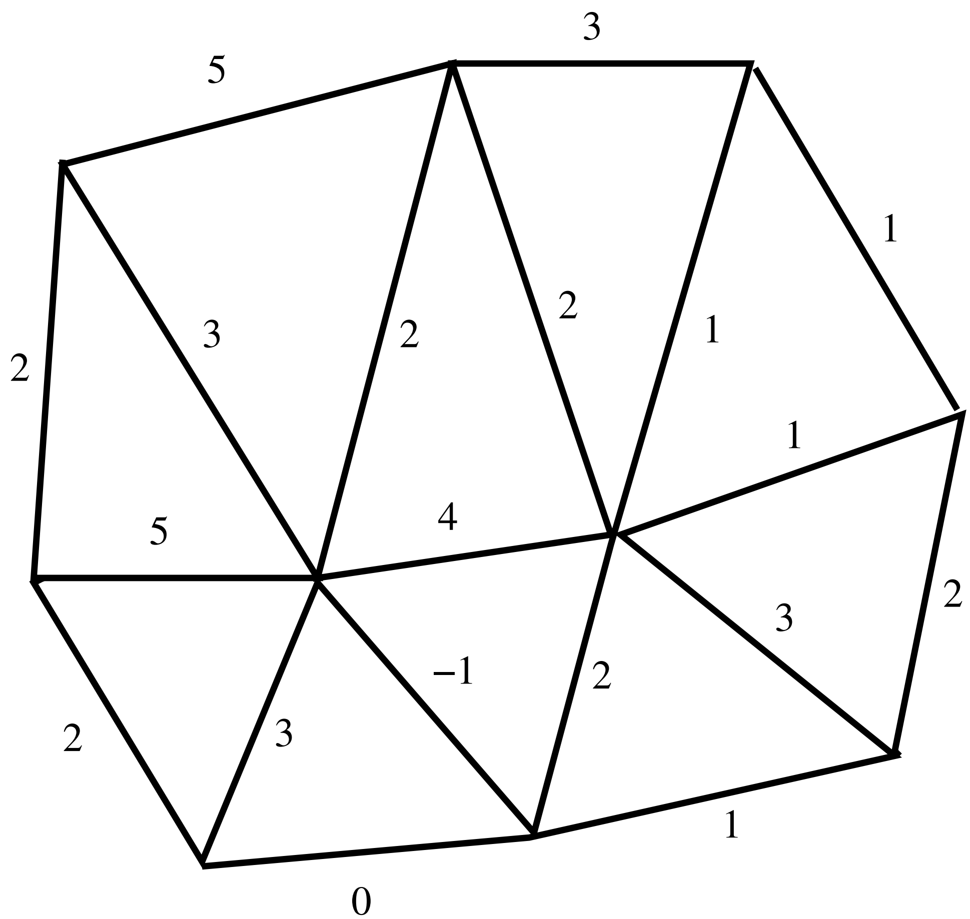
Department of Mathematical Sciences
Carnegie Mellon University

21-393 Operations Research II
Test 2

Name:____________________________________

Problem	Points	Score
1	50	
2	30	
3	20	
Total	100	

Q1: (50pts)
Find a minimum spanning tree in the following weighted graph.



Q2: (30pts) Let $\mathcal{W}$ denote the set of walks in a directed graph D. If W_1 is a walk from a to b and W_2 is a walk from b to c then $W_1 + W_2$ is the walk from a to c obtained by following W_1 and then W_2.
Let $\ell : \mathcal{W} \to \mathbb{R}$ be a real valued function defined on $\mathcal{W}$. Suppose that it has the following properties:

1. $\ell(C) \geq 0$ for any closed walk C. (A walk is closed if it begins and ends at the same vertex).
2. If W_1, W_1' are walks from a to b and W_2, W_2' are walks from b to c and $\ell(W_i') \geq \ell(W_i)$ for $i = 1, 2$ then $\ell(W_1' + W_2') \geq \ell(W_1 + W_2)$.

Consider the following algorithm: n is the number of vertices in D.
Initialise $W_{i,j} = (i, j)$ and $D_{i,j} = \ell(W_{i,j})$ for $i, j = 1, 2, \ldots, n$.

For $k = 1$ to n **Do**
  For $i = 1$ to n **Do**
    For $j = 1$ to n **Do**
    $D_{i,j} \leftarrow \min\{D_{i,j}, \ell(W_{i,k} + W_{k,j})\}$
    oD
  oD
oD

Prove that when the algorithm finishes,

$$D_{i,j} = \min\{\ell(P) : P \text{ is a path from } i \text{ to } j\}.$$

Q3: (20pts) Give an algorithm to solve the following scheduling problem. There are n jobs labelled $1, 2, \ldots, n$ that have to be processed one at a time on a single machine. There is an acyclic digraph $D = (V, A)$ such that if $(i, j) \in A$ then job j cannot be started until job i has been completed. The problem is to minimise $\max_j f_j(C_j)$ where for all j, f_j is a monotone increasing. As usual, C_j is the completion time of job j. This is distinct from its processing time p_j.