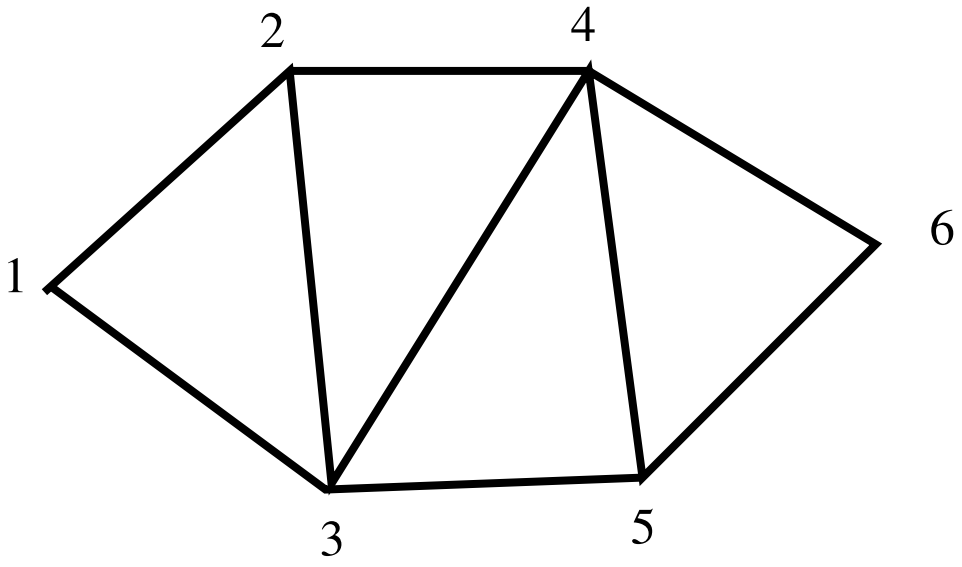
Department of Mathematical Sciences
Carnegie Mellon University

21-393 Operations Research II
Test 2

Name:_____

Problem	Points	Score
1	40	
2	40	
3	20	
Total	100	



Q1: (40pts)
Find a shortest path from vertex 1 to vertex 6 in the digraph above. The numbering of the vertices is such that every arc (i, j) is oriented from i to j when $i < j$. The arc lengths are time dependent: associated with arc $e = (i, j)$ there are two numbers, a_e, b_e such that the arc length of e is $a_e + b_e t$ where t is the time of arrival at i. The values of a_e, b_e are given in the following table:

e	(1,2)	(1,3)	(2,3)	(2,4)	(3,4)	(3,5)	(4,5)	(4,6)	(5,6)
a_e	3	4	2	5	1	3	3	4	5
b_e	2	1	3	4	1	2	1	1	3

You can put your working on the diagram.

Q2: (40pts) Given that assigning person i to job i for $i = 1, 2, 3$ is optimal for the 3×3 problem associated with the first 3 rows and columns of the matrix below, set up and solve a shortest path problem that will solve the 4×4 problem:

$$\begin{bmatrix} 0 & 3 & 2 & 4 \\ 3 & 0 & 6 & 2 \\ 3 & 5 & 0 & 3 \\ 0 & 5 & 6 & 7 \end{bmatrix}$$

Q3: (20pts)

Formulate the following problem as an integer program: A set of n items are to be repaired in a factory. Item i takes time t_i to repair and requires w_i workers working continuously. It arrives at time a_i and it must be finished by time d_i. The problem is to find a repair schedule that minimises the total number of workers needed. (When a worker has finished working on one job, he/she can work on another job).