Department of Mathematical Sciences
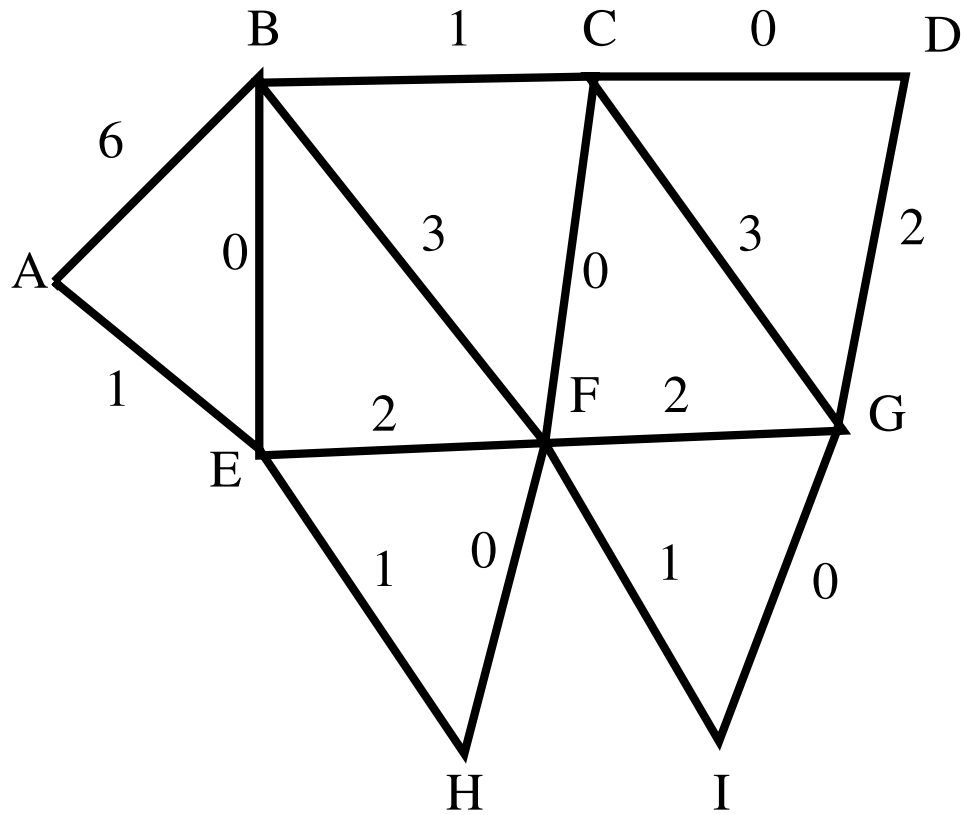# Carnegie Mellon University

21-393 Operations Research II
Test 1

Name:_____

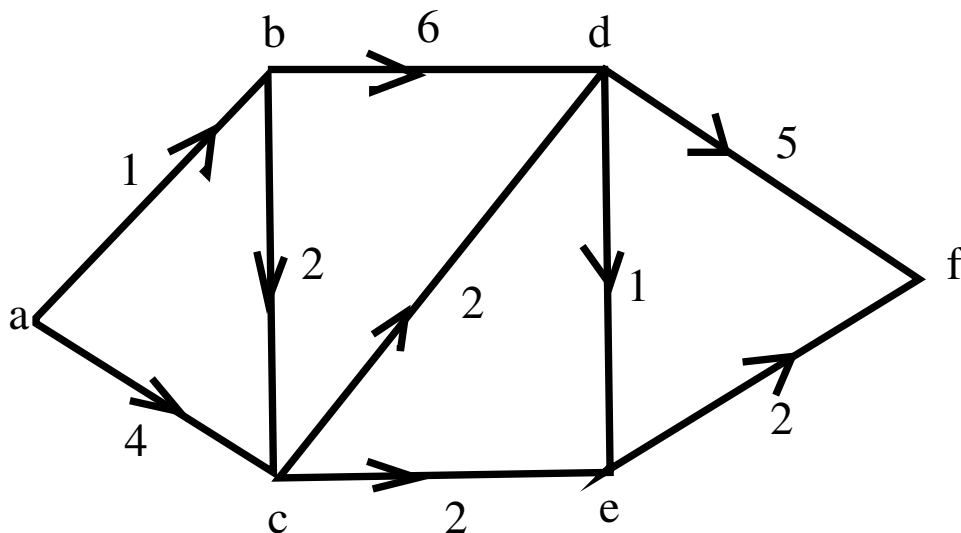| Problem	Points	Score
1	40	
2	40	
3	20	
Total	100	

Q1: (40pts)
(a) Find a minimum weight spanning tree in the graph below:

Mark your tree on the diagram.

(b) Find a shotest path from vertex a to vertex f in the graph below, using Dijkstra's algorithm:

Mark your path on the diagram.

(c) Consider the assigment problem defined by the matrix below. You are told that

$$u_1 = -1, u_2 = 0, u_3 = -2, u_4 = 0, v_1 = 2, v_2 = 2, v_3 = 4, v_4 = 2$$

is an optimal dual solution. Find an optimal assignment and use complementary slackness to show its optimality.

(You can find the assignment by trial and error if needed).

$$\begin{bmatrix} 1 & 4 & 3 & 5 \\ 3 & 2 & 5 & 2 \\ 4 & 1 & 2 & 6 \\ 2 & 2 & 4 & 6 \end{bmatrix}$$

Q2: (40pts) Find the optimal ordering strategy for the following inventory system. If you order an amount Q, it arrives immediately and the cost of the order is AQ^α for some $0 < \alpha < 1$. The inventory cost is I times M^β per period, for some $\beta > 0$, where M is the average inventory. The demand is λ units per period and no stock-outs are allowed.

Q3: (20pts) A project manager ina company is considering a portfolio of 10 large capital project investments. These investments differ in the estimated long-run profit they will generate as well in the amount of capital required. Let P_j and C_j denote the estimated profit and capital required for investment opportunity $j = 1, 2, \ldots, 10$. The total amount of capital available for these investments is Q.

Investment opportunities 3 and 4 are mutually exclusive and so are 5 and 6. Furthermore, neither 5 nor 6 can be undertaken unless at least one of 3 or 4 is undertaken. At least 2 out and at most 4 investment opportunities have to be undertaken from the set {1,2,7,8,9,10}.

The project manager wishes to select the combination of capital investments that will maximise the total esimated long-run profit subject to the restrictions described above.

Formulate this as an integer programing problem.