Department of Mathematical Sciences
Carnegie Mellon University

21-393 Operations Research II
Test 2

Name:______________________________________

Problem	Points	Score
1	33	
2	33	
3	34	
Total	100	

Q1: (33pts) Suppose that the $n \times n$ payoff matrix A of a game is non-singular. Let $\mathbf{1}$ stand for the column vector of n 1's. Suppose that $\mathbf{1}^T A \mathbf{1} \neq 0$. Let

$$V = \frac{1}{\mathbf{1}^T A \mathbf{1}}, \; p^T = V \mathbf{1}^T A^{-1}, \; q = V A^{-1} \mathbf{1}.$$

Prove that if $p, q \geq 0$ then the game has value V and that p, q are optimal strategies.

Q2: (33pts) Solve the following 2-person zero-sum games:

$$\begin{bmatrix} 7 & 2 & 3 & 1 \\ 6 & 2 & 4 & 1 \\ 4 & 3 & 4 & 4 \\ 1 & 2 & 1 & 4 \end{bmatrix} \qquad \begin{bmatrix} 7 & 4 & 4 & 1 \\ 6 & 4 & 4 & 1 \\ 4 & 1 & 4 & 4 \\ 6 & 4 & 1 & 4 \end{bmatrix} \qquad \begin{bmatrix} 3 & 2 & 0 & -1 \\ 5 & 3 & 0 & -1 \\ 3 & 2 & 1 & -1 \\ 4 & 1 & -1 & 1 \end{bmatrix}$$

Q3: (33pts) There are 3 assets with data given below:

$$V = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1/4 \\ 0 & 1/4 & 1 \end{bmatrix}, \qquad \bar{r} = \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}$$

Find 2 efficient funds F_1, F_2 for which every other efficient portfolio.can be expressed as a linear combination $\alpha F_1 + (1-\alpha)F_2$.