

Department of Mathematical Sciences
Carnegie Mellon University
21-393 Operations Research II
Test 2

Name: _____

| Problem | Points | Score |
|---------|--------|-------|
| 1 | 33 | |
| 2 | 33 | |
| 3 | 34 | |
| Total | 100 | |

Q1: (33pts) The payoff matrix A of a two-person zero-sum game has n rows and n columns and is **anti-symmetric** i.e. $A^T = -A$. Show that the game has value zero.

Q2: (33pts) Solve the following 2-person zero-sum games:

$$\begin{bmatrix} 5 & 4 & 4 & 1 \\ 6 & 5 & 5 & 2 \\ 4 & 2 & 5 & 5 \\ 6 & 5 & 2 & 5 \end{bmatrix} \quad \begin{bmatrix} 2 & 2 & 0 & -1 \\ 4 & 3 & 0 & -1 \\ 3 & 2 & 1 & -1 \\ 1 & 1 & -1 & 1 \end{bmatrix}$$

Q3: (33pts) There are 3 assets with data given below:

$$V = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1/3 \\ 0 & 1/3 & 1 \end{bmatrix}, \quad \bar{r} = \begin{bmatrix} 4 \\ 3 \\ 7 \end{bmatrix}$$

Find 2 efficient funds F_1, F_2 for which every other efficient portfolio can be expressed as a linear combination $\alpha F_1 + (1 - \alpha)F_2$.