Department of Mathematical Sciences
Carnegie Mellon University

21-393 Operations Research II
Test 2

Name:____

Problem	Points	Score
1	33	
2	33	
3	34	
Total	100	

Q1: (33pts) The playoff matrix A of a two-person zero-sum game has n rows and n columns and is **invertible**. Let $\mathbf{1}$ denote the n dimensional column vector $(1, 1, 1, 1, \ldots, 1)^T$. Suppose that

$$\mathbf{1}^T A^{-1} \mathbf{1} \neq 0 \text{ and } A^{-1}\mathbf{1} \geq 0 \text{ and } \mathbf{1}^T A^{-1} \geq 0.$$

Show, using its primal and dual Linear Programming formulations, that the game has value

$$V = \frac{1}{\mathbf{1}^T A^{-1} \mathbf{1}}$$

and optimal row and column strategies

$$p = V\mathbf{1}^T A^{-1} \text{ and } q = VA^{-1}\mathbf{1}$$

respectively.

Q2: (33pts) Solve the following 2-person zero-sum games:

$$\begin{bmatrix} 5 & 4 & 4 & 1 \\ 6 & 5 & 5 & 2 \\ 4 & 2 & 5 & 5 \\ 6 & 5 & 2 & 5 \end{bmatrix} \qquad \begin{bmatrix} 2 & 2 & 0 & -1 \\ 4 & 3 & 0 & -1 \\ 3 & 2 & 1 & -1 \\ 1 & 1 & -1 & 1 \end{bmatrix}$$

Q3: (33pts) There are 3 assets with data given below:

$$V = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1/2 \\ 0 & 1/2 & 1 \end{bmatrix}, \qquad \bar{r} = \begin{bmatrix} 3 \\ 5 \\ .8 \end{bmatrix}$$

Find 2 efficient funds F_1, F_2 for which every other efficient portfolio can be expressed as a linear combination $\alpha F_1 + (1-\alpha)F_2$.