Department of Mathematics
Carnegie Mellon University

21-393 Operatons Research II
Test 1

Name:_____

Problem	Points	Score
1	33	
2	33	
3	34	
Total	100	

Q1: (33pts)

Solve the following linear program for all values of λ:

$$
\begin{array}{llrcl}
\text{minimise} & x_1 & + & x_2 & \\
\text{subject to} & & & & \\
& x_1 & + & 2x_2 & \geq 3 - \lambda \\
& 2x_1 & - & x_2 & \geq 2 - \lambda \\
& & & & \\
& x_1, x_2 \geq 0. & & &
\end{array}
$$

[Hint: start computation with the all slack basis.]

Q2: (33pts)

Solve the following integer program:

$$\begin{array}{lrcrcrcl}
\text{maximise} & 4x_1 & + & x_2 & & \\
\text{subject to} & & & & & \\
& x_1 & + & 2x_2 & \leq & 4 \\
& 2x_1 & + & x_2 & \leq & 5 \\
\end{array}$$

$$x_1, x_2 \geq 0 \text{ and integer.}$$

Q3: (34pts)

Formulate the following as an integer program:

Suppose that a state sends R persons to the U.S. House of Representatives. There are $D > R$ counties in the state and the state legislature wants to group these counties into R distinct electoral dstricts, each of which sends a delegate to Congress. The total population of the state is P, and the legislature wants to form districts whose population approximates $p = P/R$. Suppose that the appropriate legislative committee studying the electoral districting problem generates a long list of $N > R$ *candidates* to be districts. Each of the candidates contains contiguous counties and the total population of candidate j is p_j, $j = 1, 2, \ldots, N$. Define $c_j = |p_j - p|$ and

$$a_{i,j} = \begin{cases} 1 & \text{if county } i \text{ is included in candidate } j \\ 0 & \text{otherwise} \end{cases}$$

Given the values of $c_j, a_{i,j}$, the objective is to select R of these candidates such that each county is contained in a single district and such that the largest of the associated c_j is as small as possible.