

**Department of Mathematics**  
**Carnegie Mellon University**  
21-393 Operations Research II  
Test 1

Name: \_\_\_\_\_

Problem	Points	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

**Q1: (25pts)**

Solve the following linear program by using the Upper Bounded Simplex Algorithm:

$$\begin{aligned} &\text{maximise} && 2x_1 & - & 3x_2 \\ &\text{subject to} && 23x_1 & + & 34x_2 & \leq & 837 \\ &&& 17x_1 & + & 19x_2 & \leq & 596 \\ &&& 0 \leq x_1 \leq 1, & 0 \leq x_2. \end{aligned}$$

**Q2: (25pts)**

Solve the following linear program for all values of  $\lambda$ :

$$\begin{aligned} &\text{maximise} && (\lambda - 2)x_1 & - & 3x_2 \\ &\text{subject to} && & & \\ & && x_1 & + & x_2 & \leq & 2 \\ & && -x_1 & + & x_2 & \leq & 1 \\ & && x_1, x_2 & \geq & 0. \end{aligned}$$

**Q3: (25pts)**

Solve the following integer program:

$$\text{maximise } -3x_1 - \frac{1}{2}x_2$$

subject to

$$\begin{aligned} 2x_1 + \frac{1}{2}x_2 + x_3 &= 4\frac{1}{2} \\ -x_1 + \frac{1}{2}x_2 + x_4 &= 2\frac{1}{2} \end{aligned}$$

$$x_1, x_2, x_3, x_4 \geq 0 \text{ and integer.}$$

**Q4: (25pts)**

Formulate the following as an integer program:

The Financial Aids office at Carnegie Mellon University is preparing its awards for the coming year. It has selected  $n$  students to receive awards, and wants to grant at least  $m_i$  dollars to Student  $i$ ,  $i = 1, 2, \dots, n$ . The office has  $s$  different scholarships available; Scholarship  $j$  confers the amount  $a_j$  on its recipient. The office may have to award several scholarships to an individual in order to provide the minimum it has decided that he/she will receive. The office cannot however reduce the amount of a scholarship award. If the office does not award a particular scholarship then it becomes available for next year. The office wishes to maximise the amount of money not spent in this way.