

**Department of Mathematical Sciences**  
**Carnegie Mellon University**  
21-393 Operations Research II  
Test 1

Name: \_\_\_\_\_

Problem	Points	Score
1	35	
2	35	
3	30	
Total	100	

**Q1: (35pts)**

A factory produces a single product over the next  $n$  periods. The demand in period  $i$  is  $d_i, i = 1, 2, \dots, n$  and must be met immediately. The cost of producing  $x$  items on a machine of age  $t$  is  $c_t(x)$ . The cost of repairing a machine age  $t$  so that it performs as well as a machine aged  $s$  is  $r(t, s)$ . A machine aged  $T$  or more must be replaced at a cost of  $R$ . The maximum amount that can be held in stock from one period to the next is  $H$ . Construct a recurrence that can be used to solve the problem of meeting demand at minimum total cost.

**Q2: (35pts)**

Formulate the following as an integer program: A university has  $n$  rooms available and there are  $2n$  classes  $M_1, M_2, \dots, M_n$  and  $A_1, A_2, \dots, A_n$  where  $M_1, M_2, \dots, M_n$  and  $A_1, A_2, \dots, A_n$  are both partitions of the set of students  $S$ . The classes  $M_i$  will take place in the morning and the classes  $A_i$  will take place in the afternoon. The distance between classroom  $k$  and classroom  $\ell$  is  $d_{k,\ell}$ . The problem is to assign classes to rooms in order to minimize the total distance travelled by students in changing classes.

(Hint: let  $y_{i,k,j,\ell} = 1$  iff  $M_i$  takes place in room  $k$  and  $A_j$  takes place in room  $\ell$ . It will help you to use the notation  $m_{i,j} = |M_i \cap A_j|$ .)

**Q3: (30pts)**

The simplex algorithm applied to the LP relaxation of a pure integer program results in the following tableau:

$x_1$	$x_2$	$x_3$	$x_4$	R.H.S.	
$-3/2$	0	$-5/2$	0	$17/2$	$z$
$-5/2$	0	$3/2$	1	$7/2$	$x_4$
$-1/2$	1	$1/2$	0	$5/2$	$x_2$

Finish the solution of the Integer Program using Gomory cuts. What is the optimal solution?

(One cut and one further pivot should suffice.)