Department of Mathematical Sciences
Carnegie Mellon University

21-393 Operations Research II
Test 1

Name:______________________________________

Problem	Points	Score
1	35	
2	35	
3	30	
Total	100	

Q1: (35pts)
(a) Fill in the last column of the table below for solving the following knapsack problem and produce an optimal solution:

$$\begin{array}{llllll} \text{maximise} & 3x_1 & + & 7x_2 & + & 16x_3 \\ \text{subject to} & & & & & \\ & 2x_1 & + & 3x_2 & + & 6x_3 \leq 12 \end{array}$$

$$x_1, x_2, x_3 \geq 0 \text{ and integer.}$$

w	$f_1(x_1)$	b_1	$f_2(x_2)$	b_2	$f_3(x_3)$	b_3
0	0	0	0	0		
1	0	0	0	0		
2	3	1	3	0		
3	3	1	7	1		
4	6	1	7	1		
5	6	1	10	1		
6	9	1	14	1		
7	9	1	14	1		
8	12	1	17	1		
9	12	1	21	1		
10	15	1	21	1		
11	15	1	24	1		
12	18	1	28	1		

(b) Solve the problem

$$\begin{array}{lccccccc}
\text{minimise} & 2x_1 & + & 3x_2 & + & 6x_3 & & \\
\text{subject to} & & & & & & & \\
 & 3x_1 & + & 7x_2 & + & 16x_3 & \geq & 20
\end{array}$$

$$x_1, x_2, x_3 \geq 0 \text{ and integer.}$$

Q2: (35pts)
A scout has to pack their knapsack. The knapsack has weight capacity W and there are n types of item that can be packed. Each item of type j selected provides v_j in value to the scout. The weight of an item of type j is a random variable and the scout only finds out this weight after deciding how many items of type j to take. Each item of type j does weigh the same amount. The scout does know the weight distribution for items of type j. Thus $P(w, j)$ is the probability that an item of type j weighs w. Set up a recurrence to maximise the expected value of the items that the scout can carry.

Q3: (30pts) Woody the woodcutter will cut a given log of wood of length ℓ, at any place you choose, for a price equal of $f(\ell)$, for some function $f > 0$. Suppose you have a log of length L, marked to be cut in n different locations labeled $1, 2, \ldots, n$. For simplicity, let indices 0 and $n+1$ denote the left and right endpoints of the original log of length L. Let d_i denote the distance of mark i from the left end of the log, and assume that $0 = d_0 < d_1 < d_2 < \cdots < d_n < d_{n+1} = L$. The wood-cutting problem is the problem of determining the sequence of cuts to the log that will cut the log at all the marked places and minimize your total payment. Give a dynamic programming formulation to solve this problem. Estimate the number of arithmetic operations needed by your algorithm.