Department of Mathematical Sciences
Carnegie Mellon University

21-393 Operations Research II
Test 1

Name:______________________________________

Problem	Points	Score
1	30	
2	30	
3	40	
Total	100	

Q1: (30pts)
(a) Fill in the last column of the table below for solving the following knapsack problem:

$$\begin{array}{llllllll} \text{maximise} & 3x_1 & + & 7x_2 & + & 17x_3 & & \\ \text{subject to} & & & & & & & \\ & 2x_1 & + & 3x_2 & + & 6x_3 & \leq & 12 \end{array}$$

$$x_1, x_2, x_3 \geq 0 \text{ and integer.}$$

What is the optimal solution?

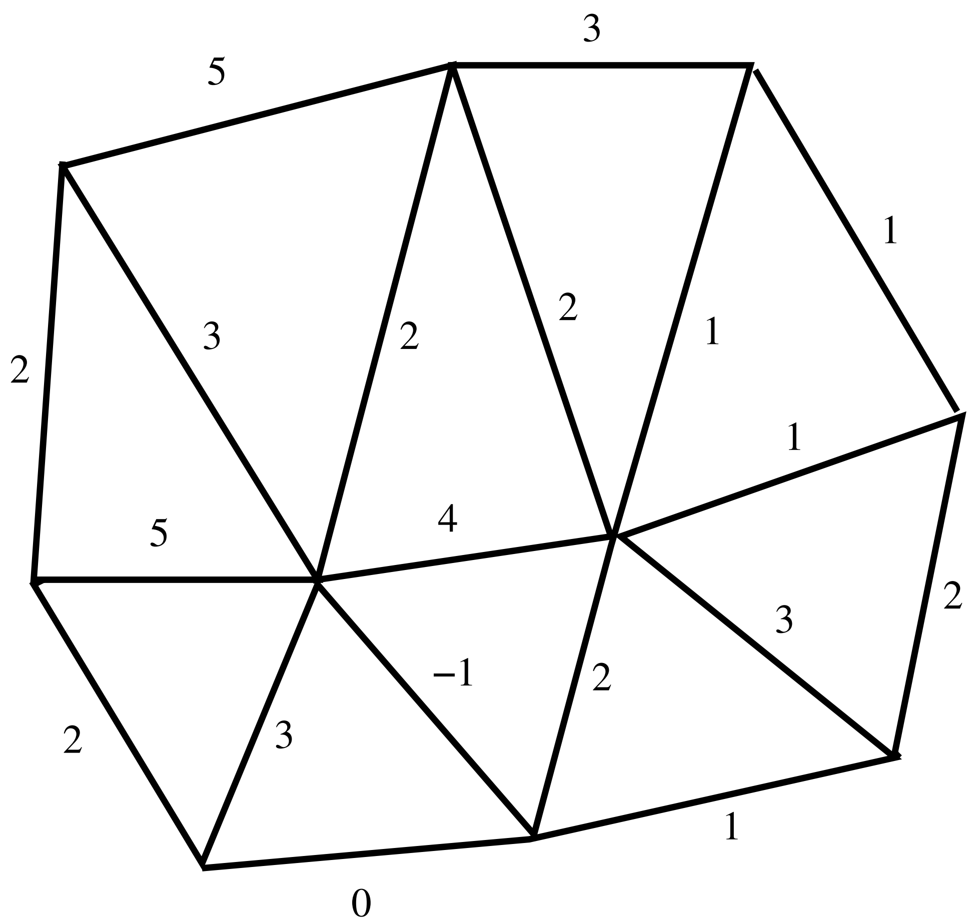
w	$f_1(x_1)$	b_1	$f_2(x_2)$	b_2	$f_3(x_3)$	b_3
0	0	0	0	0		
1	0	0	0	0		
2	3	1	3	0		
3	3	1	7	1		
4	6	1	7	1		
5	6	1	10	1		
6	9	1	14	1		
7	9	1	14	1		
8	12	1	17	1		
9	12	1	21	1		
10	15	1	21	1		
11	15	1	24	1		
12	18	1	28	1		

Q2: (30pts)

A factory uses a single machine to manufacture two distinct products A and B. It costs $c_A(x)$ to make x units of A and $c_B(x)$ to manufacture x units of B. The demand for A in period j is $d_j(A)$ and the demand for B in period j is $d_j(B)$. The factory can store at most H units altogether at any one time. Demand for A must be met in the period that it occurs, either from inventory or from production that period. Demand for B can be met in the period that it occurs, or in the following period.

Design a dynamic programming algorithm for finding the cheapest way of meeting demand for the next n periods.

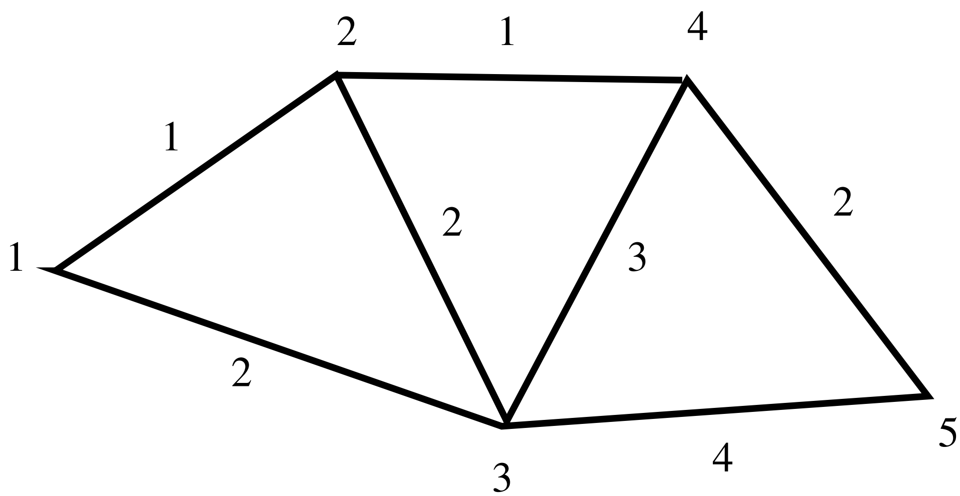
Q3: (40pts)
(a) Find a minimum spanning tree in the following weighted graph.



(b) Find a path from vertex 1 to all other vertices of the digraph below that minimises the path function ℓ. Here, if the edges have length $\ell(e), e \in E$ then a path $P = (e_1, e_2, \ldots, e_k)$ has length

$$\ell(P) = \ell(e_1) + 2\ell(e_2) + 3\ell(e_3) + \cdots + k\ell(e_k).$$

In the figure below, an edge (i, j) with $i < j$ is directed from i to j and its length is given in the middle of the edge.


2
1
4
1
2
2
1
3
2
5
3
4