Department of Mathematical Sciences
Carnegie Mellon University

21-393 Operations Research II
Test 1

Name:____

Problem	Points	Score
1	40	
2	40	
3	20	
Total	100	

Q1: (40pts)
(a) Fill in the last column of the table below for solving the following knapsack problem:

$$\begin{array}{lrcrcrcl} \text{maximise} & 3x_1 & + & 7x_2 & + & 17x_3 & & \\ \text{subject to} & & & & & & & \\ & 2x_1 & + & 3x_2 & + & 6x_3 & \leq & 10 \end{array}$$

$$x_1, x_2, x_3 \geq 0 \text{ and integer.}$$

What is the optimal solution?

w	$f_1(x_1)$	b_1	$f_2(x_2)$	b_2	$f_3(x_3)$	b_3
0	0	0	0	0		
1	0	0	0	0		
2	3	1	3	0		
3	3	1	7	1		
4	6	1	7	1		
5	6	1	10	1		
6	9	1	14	1		
7	9	1	14	1		
8	12	1	17	1		
9	12	1	21	1		
10	15	1	21	1		

Q2: (30pts)

A factory uses a single machine to manufacture two distinct products A and B. It costs $c_A(x)$ to make x units of A and $c_B(x)$ to manufacture x units of B. The demand for A in period j is $d_j(A)$ and the demand for B in period j is $d_j(B)$. If the factory makes a positive amount of both A and B in a period, then there is an extra changeover cost of K for that period. The factory can store at most H of each product. Demand must be met in the period that it occurs, either from inventory or from production that period.

Design a dynamic programming algorithm for finding the cheapest way of meeting demand for the next n periods.

Q3: (30pts)

Formulate the following problem as an integer program.

The sales area of a company is divided up into n sub-divisions $A_1, A_2, \ldots, A_n$. The company has N sales people altogether. Each salesperson allocated to A_j is expected to generate r_j dollars in revenue, but is expected to cost s_j dollars in expenses. There are at most S dollars available for expenses in the period under discussion. Sub-division A_j must be allocated at least L_j salespeople. What allocation of salespeople to districts will maximise total profit i.e. total revenue less total expenses.