Department of Mathematical Sciences
Carnegie Mellon University

21-393 Operations Research II
Test 1

Name:______________________________________

Problem	Points	Score
1	33	
2	33	
3	34	
Total	100	

Q1: (33pts)
(a) Solve the following knapsack problem, writing the results of the dynamic programming recursion in a table. You will not score any points for just writing down the answer:

$$\begin{array}{lccccccc} \text{maximise} & 3x_1 & + & 8x_2 & + & 13x_3 & & \\ \text{subject to} & & & & & & & \\ & 2x_1 & + & 3x_2 & + & 5x_3 & \leq & 10 \end{array}$$

$$x_1, x_2, x_3 \geq 0 \text{ and integer.}$$

Your answer should consist of a table.
(b) Using the answer to part (a), solve the following problem:

$$\begin{array}{lccccccc} \text{minimise} & 2x_1 & + & 3x_2 & + & 5x_3 & & \\ \text{subject to} & & & & & & & \\ & 3x_1 & + & 8x_2 & + & 13x_3 & \geq & 20 \end{array}$$

$$x_1, x_2, x_3 \geq 0 \text{ and integer.}$$

(This does not require any new computations!)

Q2: (33pts) A system can be in 3 states 1,2,3 and the cost of moving from state i to state j in one period is $c(i, j)$, where the $c(i, j)$ are given in the matrix below. The one period discount factor α is 1/2.
The aim is to find a policy which simultaneously minimises the discounted cost of operating from any starting state. Start with the policy

$$\pi(1) = 1, \pi(2) = 3, \pi(3) = 2.$$

Evaluate this policy. Is it optimal? If not find an improved policy.
YOU DO NOT NEED TO EVALUATE THIS NEW POLICY OR FIND AN OPTIMAL STRATEGY.
The matrix of costs is

$$\begin{bmatrix} 6 & 3 & 1 \\ 4 & 2 & 6 \\ 1 & 5 & 2 \end{bmatrix}$$

Q3: (34pts) You are given two strings $a = a_1, a_2, \ldots, a_n$ and $b = b_1, b_2, \ldots, b_m$ over an alphabet Σ. Here $n > m$ and we must add $n - m$ symbols to the shorter string b to make a new string $b' = b'_1, b'_2, \ldots, b'_n$. The new symbols can be placed **anywhere** within b (including before b_1 and after b_m).
For example if $n = 6$ and $m = 3$ and $b = x, x, z$ and $\Sigma = \{x, y, z\}$ we could take $b' = x, z, x, y, z, y$.
Given a, b' we have a score $s(a, b') = \sum_{i=1}^{n} f(a_i, b'_i)$ where f is some given function defined on $\Sigma \times \Sigma$.
Give a Dynamic Programming formulation for the problem of finding a b' that maximises $f(a, b')$.
(Hint: Consider the problem that remains after making a choice for b'_n.)