Department of Mathematical Sciences
Carnegie Mellon University

21-393 Operations Research II
Test 1

Name:______________________________________

Problem	Points	Score
1	33	
2	33	
3	34	
Total	100	

Q1: (33pts)

(a) Solve the following knapsack problem, writing the results of the dynamic programming recursion in a table. You will not score any points for just writing down the answer:

$$\begin{array}{lrcrcrcr} \text{maximise} & 3x_1 & + & 8x_2 & + & 14x_3 & & \\ \text{subject to} & & & & & & & \\ & 2x_1 & + & 3x_2 & + & 5x_3 & \leq & 10 \end{array}$$

$$x_1, x_2, x_3 \geq 0 \text{ and integer.}$$

Your answer should consist of a table.

(b) Using the answer to part (a), solve the following problem:

$$\begin{array}{lrcrcrcr} \text{minimise} & 2x_1 & + & 3x_2 & + & 5x_3 & & \\ \text{subject to} & & & & & & & \\ & 3x_1 & + & 8x_2 & + & 14x_3 & \geq & 20 \end{array}$$

$$x_1, x_2, x_3 \geq 0 \text{ and integer.}$$

(This does not require any new computations!)

Q2: (33pts) A system can be in 3 states 1,2,3 and the cost of moving from state i to state j in one period is $c(i,j)$, where the $c(i,j)$ are given in the matrix below. The one period discount factor α is 1/2.
The aim is to find a policy which simultaneously minimises the discounted cost of operating from any starting state. Start with the policy

$$\pi(1) = 1, \pi(2) = 1, \pi(3) = 3.$$

Evaluate this policy. Is it optimal? If not find an improved policy.
YOU DO NOT NEED TO EVALUATE THIS NEW POLICY OR FIND AN OPTIMAL STRATEGY.
The matrix of costs is

$$\begin{bmatrix} 6 & 2 & 1 \\ 4 & 2 & 6 \\ 1 & 6 & 2 \end{bmatrix}$$

Q3: (34pts)
Construct a dynamic programming functional equation that could be used to solve the following problem: A manufacture wishes to minimise the expected operating costs for the next N periods. The cost of producing x items on a machine of age t in period j, given that y items were produced in period $j-1$ is $c_j(x, y, t)$. A new machine costs an amount A and a machine of age T must be scrapped. The machine can be overhauled. It costs an amount B_i to produce a machine of equivalent age i. The purchase or overhaul of a machine can be done instantaneously at the beginning of a period. The demand d_j in period j is a random variable where $\mathbf{Pr}(d_j = d) = p_{d,j}$. It is only known *after* the decision of how much to produce in a period has been made. It must be met by the end of the next period. The maximum amount that can be held in stock for any period is H.