Department of Mathematical Sciences
Carnegie Mellon University

21-393 Operations Research II
Test 1

Name:____

Problem	Points	Score
1	33	
2	33	
3	34	
Total	100	

Q1: (33pts)
(a) Solve the following knapsack problem, writing the results of the dynamic programming recursion in a table. You will not score any points for just writing down the answer:

$$\begin{array}{llllll} \text{maximise} & 3x_1 & + & 8x_2 & + & 15x_3 & & \\ \text{subject to} & & & & & & & \\ & 2x_1 & + & 3x_2 & + & 5x_3 & \leq & 10 \end{array}$$

$$x_1, x_2, x_3 \geq 0 \text{ and integer.}$$

Your answer should consist of a table.
(b) Using the answer to part (a), solve the following problem:

$$\begin{array}{llllll} \text{minimise} & 2x_1 & + & 3x_2 & + & 5x_3 & & \\ \text{subject to} & & & & & & & \\ & 3x_1 & + & 8x_2 & + & 15x_3 & \geq & 22 \end{array}$$

$$x_1, x_2, x_3 \geq 0 \text{ and integer.}$$

(This does not require any new computations!)

Q2: (33pts) A system can be in 3 states 1,2,3 and the cost of moving from state i to state j in one period is $c(i, j)$, where the $c(i, j)$ are given in the matrix below. The one period discount factor α is 1/2.
The aim is to find a policy which simultaneously minimises the discounted cost of operating from any starting state. Start with the policy

$$\pi(1) = 1, \pi(2) = 1, \pi(3) = 2.$$

Evaluate this policy. Is it optimal? If not find an improved policy.
YOU DO NOT NEED TO EVALUATE THIS NEW POLICY OR FIND AN OPTIMAL STRATEGY.
The matrix of costs is

$$\begin{bmatrix} 6 & 2 & 1 \\ 4 & 2 & 6 \\ 1 & 6 & 2 \end{bmatrix}$$

Q3: (33pts) Dan Dare is flying his spaceship from Agmon to Zoron along the brand new Inter-Galactic Super Space Highway. The spacehip runs on Sillium fuel and there are n places along the way to stop and purchase fuel. The price at stop i is $\$p_i$ per gallon and has quality q_i. This means that each gallon will take the spaceship q_i parsecs along its journey. The spaceship has fuel capacity T and starts out with a full tank. Each re-fuelling costs f in terms of fees for entering the fuelling station. The distance from station i to staion j is $d_{i,j}$ parsecs. Assume that the start Agmon is at station 0 and the finish Zoron is at station $n + 1$. Assume also that the quality of the fuel in the tank at the start is q_0 and that adding x gallons of fuel with quality q to a tank with y gallons of quality r produces a tank with $x + y$ gallons of quality $\frac{xq+yr}{x+y}$.
Dan wishes to minimise the cost of the journey. Formulate the problem as a Dynamic Program.