Department of Mathematical Sciences
Carnegie Mellon University

21-393 Operations Research II
Test 1

Name:______________________________________

Problem	Points	Score
1	33	
2	33	
3	34	
Total	100	

Q1: (33pts) Solve the following knapsack problem, writing the results of the dynamic programming recursion in a table. You will not score any points for just writing down the answer:

$$\begin{array}{lccccccc} \text{maximise} & 3x_1 & + & 6x_2 & + & 8x_3 & & \\ \text{subject to} & & & & & & & \\ & 2x_1 & + & 3x_2 & + & 3x_3 & \leq & 10 \end{array}$$

$$x_1, x_2, x_3 \geq 0 \text{ and integer.}$$

Your answer should consist of a table.

Q2: (33pts) Formulate a dynamic programming functional equation that could be used to solve the following problem: A manufacture wishes to minimise the operating costs for the next N periods. The cost of producing x items in period j, given that y items were produced in period $j-1$ is $c_j(x, y)$. The demand in period j is d_j and it must be met. The maximum amount that can be held in stock for any period is H.

Q3: (34pts) A taxi driver's territory comprises 2 towns A,B. He has 2 alternatives:

1. He can cruise in the hope of picking up a passenger by being hailed.

2. He can drive to the nearest cab stand and wait in line.

The transition probabilities and the rewards for being in the various towns and making the various transitions are as follows:
A:

$$P = \begin{bmatrix} .25 & .75 \\ .75 & .25 \end{bmatrix} \; R = \begin{bmatrix} 10 \\ 18 \end{bmatrix}$$

B:

$$P = \begin{bmatrix} .25 & .75 \\ .25 & .75 \end{bmatrix} \; R = \begin{bmatrix} 6 \\ 15 \end{bmatrix}$$

Thus if he is in A and he drives to the nearest cab stand (alternative 2) then with probability .25 he stays in A and with probability .75 he goes to B. His expected gain for the period is 8.
His current policy is to always cruise. Use the policy iteration algorithm to find a policy that gives a better (=larger) long term average gain per period. Here you need to (i) evaluate the current policy and then (ii) find the better one, **WITHOUT EVALUATING THE SECOND ONE**.