Variation of Stable Marriage Algorithm and Its Usage in CMU Greek Life

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Introduction:

Many college campuses across the country provide students with the opportunity to "go Greek" or join organizations known as fraternities and sororities. These organizations, whose literal names are derived from Greek and Latin roots meaning brotherhood (or sisterhood), are designed to offer their members various leadership and service opportunities while fostering personal development and creating a professional network. Here at Carnegie Mellon, about 20% of the undergraduate population is involved with Greek life and the all-Greek average QPA is higher than the overall CMU average. Membership in these groups is based on a recruitment process and often varies between fraternities and sororities. There are 5 sororities at CMU and they all utilize Formal Membership Recruitment (FMR). The FMR process allows prospective members to meet and mingle with current members of different sorority chapters before listing their preferences. As a group, members of respective chapters submit their own preferences of to whom they would like to extend membership and a software system called Interactive Collegiate Solutions: Collegiate Recruiter (ICS) is used to create matches. While the ideal scenario would guarantee that each potential member receives an invitation from her top choice, in order to maintain a sense of fairness and balance between the chapters, Carnegie Mellon utilizes a "quota" concept in order to try and make sure that all 5 organizations are growing and prospering at a similar rate. The ultimate goal of the FMR process is to maximize the number of women who receive an invitation of membership from their first choice sorority while adhering as closely as possible to the quota standards set for the chapters.

The General Problem:

The general algorithm believed to motivate the ICS software and logistics of the FMR process is known to solve the stable marriage problem. This algorithm solves the following problem, stated in laymen's terms:

"Given *n* men and *n* women, where each person has ranked all members of the opposite sex with a unique number from 1 to *n* in order of preference, is it possible to marry the men and women together such that there are no two people of the opposite sex who would prefer each other over their current partners?" (Wikipedia)

The most common algorithm known to solve the above problem is the Gale-Shapley algorithm, developed by David Gale and Lloyd Shapley in 1962. The algorithm is iterative and the details of the iterations are listed below.

Gale-Shapley Algorithm Iterations:

1. Each "unengaged" man proposes to the woman ranked highest on his preference list and to whom he has not already proposed.
2. Each woman who receives a proposal becomes "engaged" to the man ranked highest on her preference list out of all of her suitors.
    i. If a woman was already "engaged" and receives a proposal from a man who ranks higher on her preference list than her current "fiancé", she can dissolve her current engagement and enter into a new one. Her previous fiancé is then considered "unengaged" and can continue with his proposals.
3. Once the matchings have been made, the process reiterates with the remaining "unengaged" men continuing through their list of proposals.

The Gale-Shapley algorithm is considered effective because it guarantees two very important details: 1) every man and woman eventually ends up engaged and 2) at the end, every engagement is stable. That is, there are no two people who would consider breaking their current engagements to be with each other. However, this does not necessarily mean that each person is matched with their top preference. The following illustrates an example of the stable marriage problem, with *n* = 3, that is solved using Gale-Shapley.

	A	B	C		**X**	**Y**	**Z**
1st choice	Z	Z	Y		C	A	A
2nd choice	X	Y	X		B	C	B
3rd choice	Y	X	Z		A	B	C

Table 1: List of preferences; **bold** indicates male

Iteration 1:
-X proposes to C; C accepts and they become engaged➔ (X,C)
-Both Y and Z proposed to A; A prefers Z over Y and thus accepts Z's proposal➔ (Z,A)
-Y remains unengaged; B received no proposals and therefore also remains unengaged

Iteration 2:
-Y proposes to next on list, C; C is engaged to X, but prefers Y over X so breaks up with X and becomes engaged to Y➔ (Y,C)
-A receives no more proposals and therefore remains with Z
-Now X is unengaged, as is B

Iteration 3:
-X proposes to next on list, B; B accepts➔ (X,B)
-There are no more unengaged males and thus the algorithm is complete.

{(X,B), (Y,C), (Z,A)} is the resulting set of stable matchings. Pairs are disjoint and no couple has incentive to break up, verifying the fact that the Gale-Shapley algorithm is effective. By this point, it is fairly obvious how the stable marriage algorithm can be related to Carnegie Mellon sorority recruitment; as mentioned in the Introduction, the process is dependent upon mutual selection and the matching of preferences. However, important discrepancies to note include the fact that the number of sorority chapters and potential members is not the same, meaning that the problem is no longer mimics the matching up of *n* men and *n* women. In addition, CMU tries to maintain a quota of sorts when extending invitations of sorority membership to potentials—this is also not accounted for within the stable marriage problem or the Gale-Shapley algorithm, making the decision to use them in practice on CMU's campus uncertain. Finally, CMU sorority recruitment focuses on maximizing the number of potential members who are ultimately matched and receive an invitation to membership from their first choice sorority, which is not necessarily the intent of the stable marriage problem. The Gale-Shapley algorithm is ideal only for the stable marriage problem and its use in this application may not provide optimal results. An algorithm that solves a variation of the problem, however, that accounts for these mentioned factors, should be considered instead.

The Variation and Application:

A common variation of the stable marriage problem that is more applicable when analyzing the FMR process is the hospitals/residents problem (also known as the college admissions problem). One of the major differences in this problem is that woman can accept "proposals" from multiple men—in reality, hospitals can accept multiple residents. Like before, both hospitals and residents make a list of preferences of the other. However, unlike before, a resident is not required to include all hospitals on his/her list; a hospital not being included indicates that a resident would prefer not being matched than being matched to that specific hospital. The overall goal is match up hospitals and residents while making sure that 1) each resident is matched or each hospital is filled and 2) each matching is stable (no resident would prefer another hospital—that had a vacancy and preferred said resident over others—over his/her current hospital). While the Gale-Shapley algorithm cannot directly be used to solve this problem, the National Resident Matching Program (NRMP) was created in 1952 to assist with the matching process and an algorithm was eventually developed. The steps of the algorithm's iteration are listed below.

National Resident Matching Program Algorithm:

1. Each unmatched resident "proposes" to the hospital program ranked highest on his/her list.
2. Hospitals accept the "proposals" if the residents also appear on their lists and there are vacancies available.
    i. A hospital can also accept a "proposal" if the resident appears on its list but there are no vacancies left. In this case, the hospital's least preferred resident who has already been matched would be let go and the new resident would receive the position.
3. Once the matchings have been made, the process reiterates, with the remaining unmatched residents continuing down their lists.
    i. However, a recently unmatched resident (aka one whose match with a hospital was recently dissolved) can start from the beginning and "propose" to his/her first choice hospital program.

The process continues until all hospitals have considered all residents; at this point, it is possible for a resident to not be matched with any hospital. However, all of the matchings that are formed at that point are stable.

The setup and overall concept of the hospital/residents problem mimics that of CMU's FMR process. In this case, the sorority chapters can be thought of as the hospitals and the potential new members going through recruitment are the residents. The entire process spans 5 nights; the first night, potential members spend some time at each of the 5 sorority chapters. At the end of the night, prospective members rank 4 chapters as their "top" choice—essentially chapters that they would be interested in learning more about and potentially joining. Meanwhile, chapter members also formulate a list of girls who they would like to see return. ICS, the software program mentioned in the introduction, is then used to run an algorithm similar to the NRMP/stable marriage algorithms in order to create matchings. The next night, potential new members can attend up to 4 chapters before creating a list of their "top" 3 while chapter members continue to refine their owns lists. This continues until potential members are left with 2 sorority chapters, one which they officially rank 1st, and the other 2nd. Also at this point, chapter members must come up with a final list of girls to whom they would like to extend an invitation of membership. ICS runs the NRMP algorithm one final time to determine the invitation status of potential members, while taking into account the set "quota". As was mentioned, CMU Greek Life utilizes a "quota" system in an attempt to maintain a balance amongst the different chapters. "Quota" is considered to be the maximum number of invitations of membership a chapter can extend and is often calculated by taking the average number of girls still in the FMR process by the final night and then flexed up or down in order to maximize placement. By the end of the process, a potential new member can either 1) receive an invitation of membership from her top choice, 2) receive an invitation of membership from her 2nd choice, or 3) not receive invitation from either choice.

Data & Analysis:

	Fall 2010	Fall 2009	Fall 2008	Fall 2007	Fall 2006	Fall 2005	Fall 2004
Total Registered	206	181	158	145	140	186	156
Total Eligible (Due to various factors)	206	181	153	145	136	178	156
Number	173	144	93	105	97	89	103

Attending Final Night							
Percentage Attending Final Night	**83.98%**	**79.56%**	**58.90%**	**72.41%**	**69.29%**	**47.85%**	**66.03%**
Withdrawn After Final Night	9	14	0	9	1	3	0
Number Accepting Invitation	164	130	93	95	95	86	103
Percentage Accepting Invitation	**79.61%**	**71.82%**	**58.90%**	**65.52%**	**97.94%**	**96.63%**	**100%**
Single Preference	1	0	1	N/A	0	1	10
Matched in Invitation Matching	164	130	93	94	94	83	103
Number Not Accepting Invitations	15	10	4	0	3	1	0
Number Receiving First Choice	141	112	82	86	80	70	101
Percentage Receiving First Choice	**86%**	**87%**	**88.17%**	**91.50%**	**85.11%**	**81.39%**	**98%**
Number Receiving Second Choice	23	18	11	8	14	12	2
Percentage Receiving Second Choice	**14%**	**13%**	**11.83%**	**8.42%**	**14.89%**	**13.95%**	**0.02%**
Not Matched	**0**	**0**	**0**	**1**	**1**	**3**	**0**
Quota	**30**	**24**	**21**	**20**	**23**	**25**	**28**
Chapters Taking Quota	5	4	4	4	4	2	2
Total Number Withdrawn & Dropped	41	51	62	33	42	26	27
Total Percentage of All Potential New Members Pledged	85.42%	74.03%	56.32%	64.82%	67.14%	44.62%	66%

Table 2: CMU Recruitment Data from Fall 2004-Fall 2010

Table 2 shows the recruitment statistics from 2004 to 2010. First row shows the total number of people who registered for recruitment and total eligible shows the number of people who meet the QPA requirement. Even though there may people who are not eligible, the CMU Greek Community encourages women to attend recruitment. Women in this position can explain their specific situation to the chapters and it is chapter’s discretion to make exceptions for women who are not eligible. The number of women who attend the final night is sometimes less than the number of women registered. Potential members have the option to leave during the recruitment process for any reason. After the final night, historically most – if not all – women will receive an invitation from a chapter. They can choose to decline the invitation if they do not receive one from their first choice chapter or for any other reasons. However, usually over 80% of the women receive their first choice chapter and less than 15% who do not receive their first choice preference. Single Preference means that the women only wish to receive an invitation from their first choice chapter. In this case, if a woman did not receive an invitation from her first choice, then she will not be placed into a chapter. Single Preference means that a woman is not maximizing her options, which means that she has a lower chance of being placed into a chapter of her choice. There are very few to no women who do this. "Quota", as mentioned earlier, is usually number of women who attend the final night divided by number of chapters. This is the maximum number of women to whom a chapter can extend invitations. However, sometimes chapters can either exceed or fall below the quota in order to match every woman with a chapter. The numbers of women who become members of the chapters are high. It was 85.42% for 2010 and this percentage has been steadily increasing from previous years.

Potential Problems & Solutions:

The first issue in the application of this algorithm to the sorority recruitment process is that the above algorithm does not account for selectivity. For example although all organizations may follow the same process for submitting their preferences, there may be different criteria upon which they construct their invitation lists. Examples of this for sororities might be that an organization could have a certain GPA cutoff, or they might set limits to how many members of a certain academic year they would like to take because perhaps it is not in the organization's interest to have many new members who are juniors or seniors. Based on these selectivity criteria, it may not be possible for some potential members to receive an invitation for their first choice house, which is a flaw in our application of this algorithm. We would want to explore adding a selectivity parameter that restricts the available matches and therefore does not disrupt the algorithm's efficiency. An option for dealing with this could be to discount the ranking if a girl does not match a chapter's selectivity criteria, independent of her preferencing. For example, suppose a girl ranks a chapter highly on her preference list, however she does not meet that chapter's criteria for some selectivity measure and therefore is placed low on the chapter's invitation list. Such a situation would be difficult for the current algorithm to deal with and would probably result in an "unsuccessful" match, as either the chapter or girl will be unhappy with the outcome.

We could potentially correct for this in our use of the algorithm by adding selectivity parameters to each girl's preferences. For example, for each chapter that a girl ranks, we will also have measures of how closely she meets that chapter's selectivity criteria. For example, suppose $0 < \alpha < 1$ measures how well the potential member fits a chapter's minimum GPA cutoff. A score of zero would indicate that they are below the cutoff, perhaps a score of 0.5 might mean they are exactly at the cutoff and there is some risk that they could fall below it, and a score close to 1 would indicate that they are exceeding the criteria. We would then multiply the potential member's preferences by this parameter before inputting into the algorithm, thus discounting the preferences of those who do not meet the chapter's selectivity criteria and maintaining the algorithm's efficiency.

We can further develop this as an operations research problem by assigning a value between 0 and 1 to each match as a measure of its optimality. We will consider a chapter's top choice members to be the highest ranked on their list up to the number that has been determined by quota. As a reminder, quota is set by dividing the total number of potential members by the number of chapters. If we suppose for example that quota is set to 25, then if a potential member is ranked with their top choice and they also fall in that chapter's top 25 ranking, then that match will assign a value of 1. Other matches will receive a value less than one if they fall below the chapter's top 25 ranking or if a potential member is matched with a second choice chapter. We want our algorithm to maximize the sum of the values over all the matches, where the most optimal would be if every match had a value of one, thus the highest possible value would be equal to the total number of potential members.

Another potential problem with the application of this algorithm to the sorority recruitment process is that it is dependent on the condition that organizations and potential members maximize their options by filling their preference lists to capacity. For example our process does allow for girls to choose to not accept a chapter's invitation and thus not preference them, as well as it does allow for chapters to leave girls entirely off their lists who they would not want to return, as opposed to keeping them on their list but ranking them low. However the algorithm is not able to maximize the number of possible matches if either potential members or chapters do not fill their lists to capacity by ranking their entire list of options. The concept of mutual selection is invalidated if a potential member or chapter leaves a chapter or potential member off their list completely and also lessens their overall chances of being matched successfully. For example, suppose a potential member does not a rank a chapter, but she appears low on the invitation list for the chapter she has preferenced, and the chapter also has been preferenced by the potential members appearing higher on their list. Due to a limitation on the number of potential members who can receive invitations from a particular chapter, this would most likely result in the potential member low on the list being entirely unmatched and the algorithm being ineffective, as opposed to the alternative situation where should would be matched with her second choice. The counterpart situation would be if a chapter chooses to eliminate girls they would not want to return from their list altogether, as opposed to keeping them on the list but ranking them low. While this may seem like an appropriate choice if the chapter has decided the potential member is not a good match, there is also risk that the potential members that they have invited back will not preference them, and that the chapter will be unmatched with the potential members who are appearing on their list, thus limiting the number of possible members they could have and increasing the number of unmatched potential members.

Conclusion:

In conclusion, we believe that the stable marriage problem and the national residents matching program algorithm have application to Carnegie Mellon's sorority recruitment matching problem. We explore this topic as it is relevant to the strong presence of Greek Life on our campus, and thus it of interest to make the process as successful as possible. We explored the application of this algorithm to the formal membership recruitment process for matching potential new members to sororities, and have found that it could be effective with some additional modifications. Some corrections we would want to introduce for the algorithm include a parameter measure to account for different selectivity criteria, and a means of determining the success of a match and maximizing the sum of these values over all of the potential new members in our sample. The goal of our problem would be to best match potential new members to chapters based on the preferencing of both the members and chapters while maintain "quota", and based on our research we believe that this algorithm with some additional modifications could be efficiently applied to this problem and that its use would increase the number of desirable matches for both parties.

Sources

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