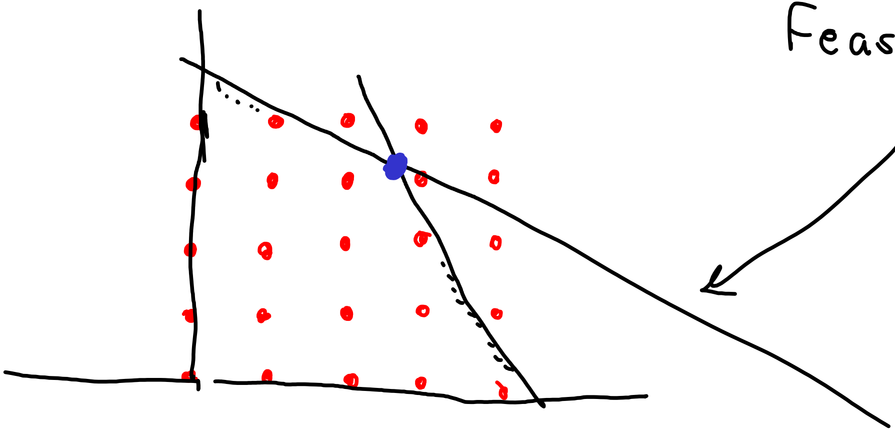
9\16\13

Cutting Plane Algorithm

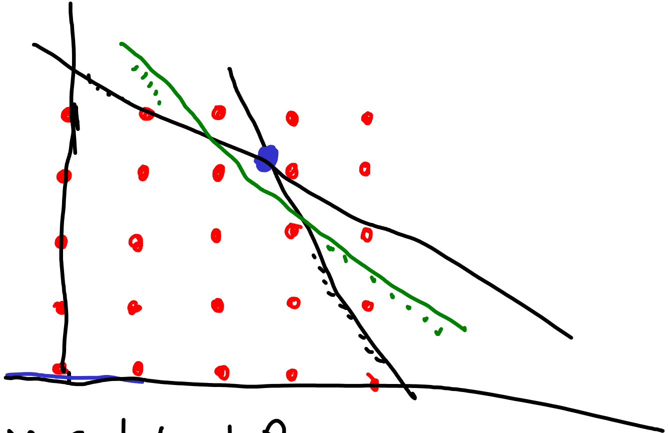


Feasible region

(1) Solve LP (ignore integrality

(a) optimum is integer - done

(b) optimum is not integer



Optimum is not an integer

Add a Cut

A cut is an inequality:

(i) Current optimum does not satisfy (it)

(ii) Every integer solution does.

Now re-solve LP & repeat until optimum is integral.

Goal is to define cuts so that the process finishes in a finite number of steps.

GOMORY CUTS

FOR THE <u>PURE</u> PROBLEM

ALL VARIABLES ARE INTEGER VARIABLES

Suppose we have solved the LP (by the simplex algorithm) and there is non-integer basic variable

There's an equation

All non-neg. integer feasible solutions satisfy

$$\sum_{j \in N} b_{ij} x_j + x_i = b_{i0}$$

non-basic variables

basic variable

not an integer.

In general suppose we have the equation

$$a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = b$$

$S \subseteq \mathbb{R}^n$ is the set of non-negative integer solutions to

$$a_j = \lfloor a_j \rfloor + f_j \quad \text{and} \quad b = \lfloor b \rfloor + f$$

$\lfloor x \rfloor$ = largest integer $\leq x$.

$$\lfloor 3\tfrac{1}{2} \rfloor = 3$$

$$\lfloor -3\tfrac{1}{2} \rfloor = -4$$

$$\sum_{j=1}^{n} (\lfloor a_j \rfloor + f_j) x_j = \lfloor b \rfloor + f$$

Assume $f > 0$

$$\underbrace{\lfloor b \rfloor - \sum_{j=1}^{n} \lfloor a_j \rfloor x_j}_{\text{integer}} = \underbrace{\sum_{j=1}^{n} f_j x_j - f}_{\text{integer}} \geq 0$$

Assume $(x_1, \ldots, x_n) \in S$

integer & ≥ 0

$\geq -f > -1$

$$\sum_{j \in N} b_{ij} x_j + x_i = b_{i0}$$

$$\Downarrow$$

$$\sum_{j \in N} f_j x_j \geq f \underset{\sim\sim}{>0} \quad \text{Gomory Cut}$$

The current optimum has $x_j = 0$, $j \in N$ and does not satisfy

Maximise $3x_1 + 2x_2$

$$2x_1 + 3x_2 \leq 7$$

$$4x_1 + 3x_2 \leq 11$$

$x_1, x_2 \geq 0$ & integer

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B.V.	x_1	x_2	x_3	x_4	RHS
x_0	-3	-2			0
x_3	2	3	1		7
x_4	4*	3		1	11

B.V.	x_1	x_2	x_3	x_4	RHS
x_0		$1/4$		$3/4$	$33/4$
x_3		$1/2$	1	$-1/2$	$3/2$
x_1	1	$3/4$		$1/4$	$11/4$

$$\tfrac{1}{4}x_2 + \tfrac{3}{4}x_4 \geq \tfrac{1}{4}$$

B.V.

X_0

X_3

X_1