9\11\13

$$C = \begin{bmatrix} 3 & 2 & 1 & 5 \\ 4 & 3 & 4 & 1 \\ 2 & 4 & 1 & 3 \\ 5 & 4 & 3 & 4 \end{bmatrix}$$

i	1	2	3	4
$\pi(i)$	4	3	2	1
y_i	10	8	8	10

$\underline{i=1}$

$c_{ij} + \alpha y_j$

$3 + \frac{1}{2} 10$

$2 + \frac{1}{2} 8$

$1 + \frac{1}{2} 8 \leftarrow 5$ *

$5 + \frac{1}{2} \times 10$

$$C = \begin{bmatrix} 3 & 2 & 1 & 5 \\ 4 & 3 & 4 & 1 \\ 2 & 4 & 1 & 3 \\ 5 & 4 & 3 & 4 \end{bmatrix}$$

$\underline{i = 2}$

$4 + \frac{1}{2} \times 10$

$3 + \frac{1}{2} \times 8$

$4 + \frac{1}{2} \times 8$

$1 + \frac{1}{2} \times 10 = 6$

$\underline{i = 3}$

$2 + \frac{1}{2} \times 10$

$4 + \frac{1}{2} \times 8$

$1 + \frac{1}{2} \times 8 = 5$

$3 + \frac{1}{2} \times 10$

$\underline{i = 4}$

$5 + \frac{1}{2} \times 10$

$4 + \frac{1}{2} \times 8$

$3 + \frac{1}{2} \times 8 \quad 7$

$4 + \frac{1}{2} \times 10$

$\underline{\text{New } \pi}$

3 4 3 3

Evaluate

$$y_1 = 1 + \tfrac{1}{2} y_3 = 2$$

$$y_2 = 1 + \tfrac{1}{2} y_4 = 2$$

$$y_3 = 1 + \tfrac{1}{2} y_3 = 2$$

$$y_4 = 3 + \tfrac{1}{2} y_3 = 4$$

$$C = \begin{bmatrix} 3 & 2 & 1 & 5 \\ 4 & 3 & 4 & 1 \\ 2 & 4 & 1 & 3 \\ 5 & 4 & 3 & 4 \end{bmatrix}$$

Then

Check for optimality

⋮

Optimality Conditions

(1) Assume π satisfies optimality conditions.

$\hat{\pi}$ is any other policy.

$$\xi_i = \hat{y}_i - y_i$$

$$\hat{y}_i = c_{i,\hat{\pi}(i)} + \alpha\, y_{\hat{\pi}(i)}$$

$$y_i \leq c_{i,\hat{\pi}(i)} + \alpha\, y_{\hat{\pi}(i)}$$

$$\Rightarrow \quad \xi_i \geq \alpha\, \xi_{\hat{\pi}(i)}$$

$$\xi_i \geq \alpha\, \xi_{\hat{\pi}(i)} \geq \alpha^2 \xi_{\hat{\pi}^2(i)} \geq \cdots \geq \alpha^k \xi_{\hat{\pi}^k(i)} \to 0$$

$$\Rightarrow \xi_i \geq 0, \quad \forall i$$

So, if optimality condition holds, we have best y_i, $\forall i$

(ii) Suppose optimality condition does not hold.

$y_i > c_{ij} + \alpha y_j$ for some i, j

So define $\hat{\pi}$ by

$\hat{\pi}(i) = \underset{j}{\operatorname{argmin}}\ c_{ij} + \alpha y_j$ & $\hat{\pi} \neq \pi$

which minimises

$I = \{ i : \pi(i) \neq \hat{\pi}(i) \}$

To show (i) $\hat{y}_i \leq y_i$ & (ii) $i \in I \Rightarrow \hat{y}_i < y_i$

$\Longrightarrow$ no repetition of π

& eventual termination with

optimality condition.

$$\hat{y}_i = c_{i,\hat{\pi}(i)} + \alpha\, y_{\hat{\pi}(i)}$$

$$y_i \geq c_{i,\hat{\pi}(i)} + \alpha\, y_{\hat{\pi}(i)} \quad \Big\} \; \forall\, i \in I$$

$\xi_i \leq \hat{y}_i - y_i$ then

$$\xi_i \leq \alpha\, \xi_{\hat{\pi}(i)} \leq \cdots \leq \alpha^k \xi_{\hat{\pi}^k(i)} \to 0 \quad \Big\} \text{ if } i \in I$$

Solution via Linear Programming.

We have to find $y_1, y_2, \dots, y_n$

s.t.

$$y_i = \min_j \Big[\underbrace{C_{ij} + \alpha y_j} \Big]$$

then $\pi(i) = \text{argmin}$

maximise

$y_1 + y_2 + \cdots + y_n$

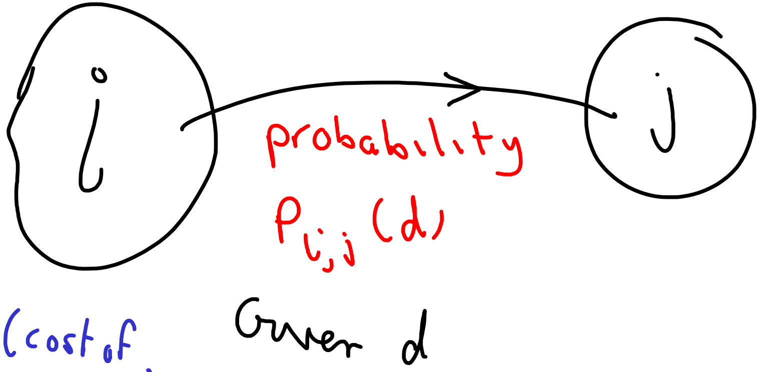
subject to

$$y_i \leq c_{ij} + \alpha y_j$$

$\forall i, j$

Probabilistic Version

$D_i = \{$possible decisions e.g. how much to produce$\}$



$\pi(i) \in D_i, \forall i$

Choose π to minimise expected NPV.

$c_{i:d}$ = E(cost of move)