

10/2/13

All pairs shortest path problem

Given a matrix $D = \|d_{ij}\|$ where d_{ij} = length of arc (i,j) .

Problem: find a shortest walk/path from i to j .

Floyd's Algorithm

$$\underline{D_{ij} \leftarrow d_{ij}}$$

$k=0$

For $k=1$ to n do

For $i=1$ to n do

For $j=1$ to n do

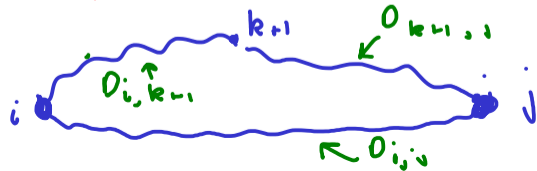
$$D_{ij} \leftarrow \min \{ D_{ij}, D_{ik} + D_{kj} \}$$

D
Z
C
O
R

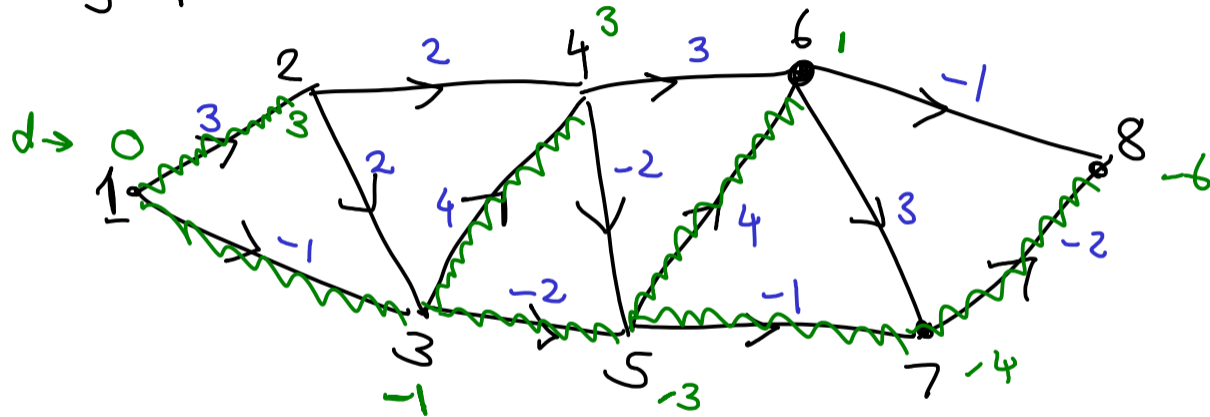
od
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Claim: After k rounds
 D_{ij} = min. length of a walk
from i to j , where internal
vertices are from $1, 2, \dots, k$

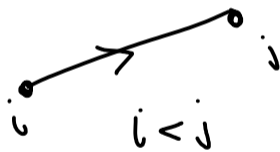
Proof: Induction on k
 $k=0$: by initialisation.
Assume true for k



Digraphs without directed cycles DAG



Topological ordering:

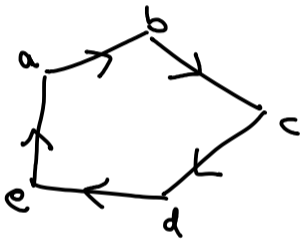


Optimality condition: $d(j) = \min_{k < j} [d(k) + l(k, j)]$

Claim: \exists a topological ordering iff there are no directed cycles. [Finite # of vertices = n]

Proof

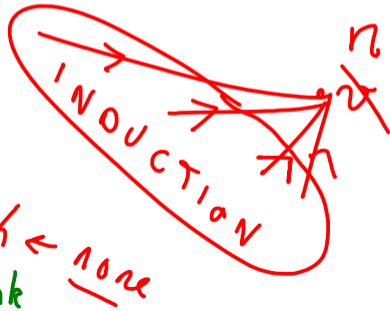
\exists ordering:



$a < b < c < d < e < a$?

~~cycles~~: (a) \exists a vertex v with no outgoing edges - sink

Take longest path

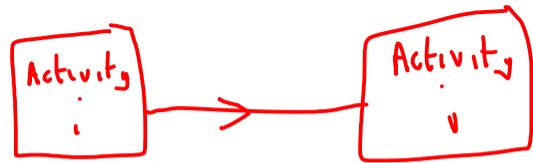


Critical Path Analysis

Large job = { activities }

Make Coffee

- 1 Get cups
- 2 Boil Water
- 3 Get Milk
- 4 Put Milk in cups
- 5 Pour in water
- 6 Stir



Do i before j

