10\25\13

# Dominance

row 1 dominates row 2

col 5 dominates columns 3 & 4

$$\begin{bmatrix} 6 & 5 & 3 & 4 & 1 \\ 3 & 2 & 2 & 3 & 0 \\ 2 & 1 & 5 & 7 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 6 & 5 & 1 \\ 2 & 1 & 4 \end{bmatrix}$$

col 2 dominates col 1

## Special Cases

Symmetric Case: $A^T = -A$

$$\begin{array}{c|ccc} & R & P & S \\ \hline R & 0 & -1 & +1 \\ P & +1 & 0 & -1 \\ S & -1 & +1 & 0 \end{array}$$

Claim: In stable solution $p = q$.

$p^T A p = 0, \ \forall p \qquad \sum_{i,j} a_{ij} p_i p_j$

$$= \sum_{i<j} (a_{ij} + a_{ji}) p_i p_j$$

$$= 0$$

$PAY(p,q)$
$$\begin{bmatrix} 0 & & & & & \\ & 0 & & & & \\ & & 0 & & & \\ & & & 0 & & \\ & & & & 0 & \\ & & & & & 0 \end{bmatrix}$$

$p_A \le 0$

$p_B \ge 0$ & $p_A = p_B$

$\Downarrow$

$p_A = p_B = 0$

# Approximation Algorithms

TSP $\quad c_{ij} + c_{jk} \geq c_{ik} \geq 0$ triangular inequality

① 2-approximation.
Produces a tour $T$ such

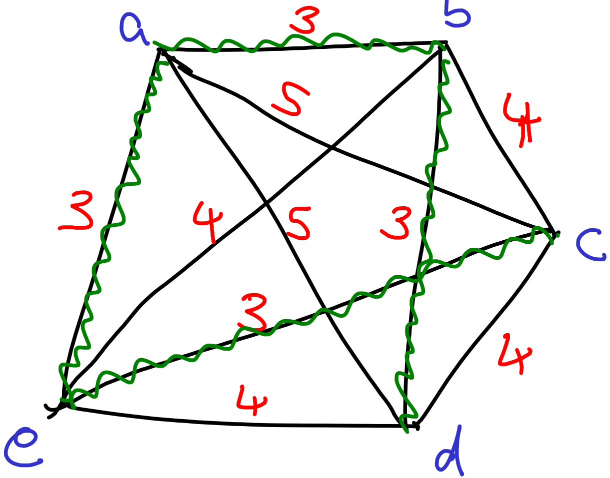
$$l(T) \leq 2\, l(\underset{\text{optimum}}{T^*})$$

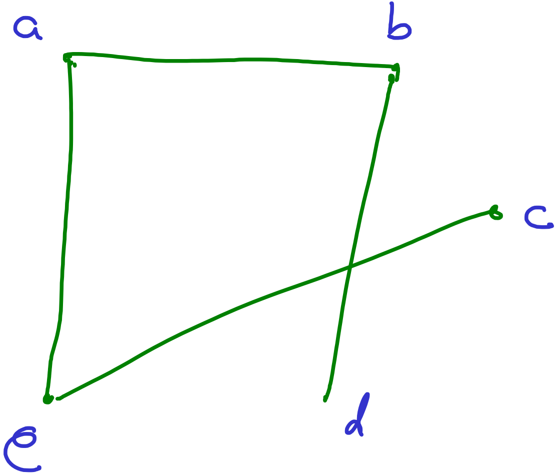
(a) Find a minimum length spanning tree

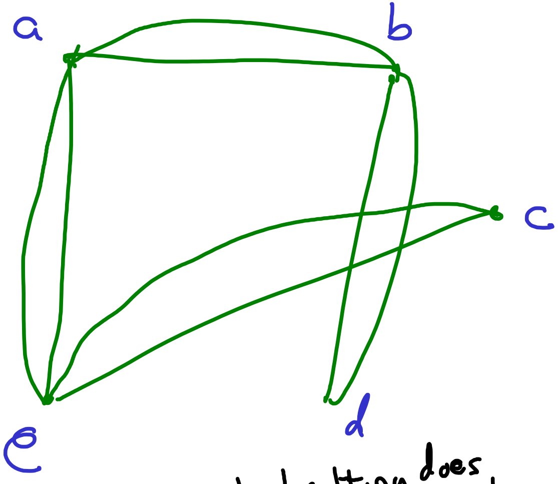
(b) Double the edges.

(c) Euler tour

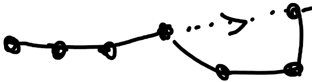
(d) short cut


a
b
c
d
e
3
5
4
3
4
5
3
3
4
4


a
b
c
d
e




shortcutting does not make tour longer



Euler tour $T$

d, b, a, e, c, e, a, b, d

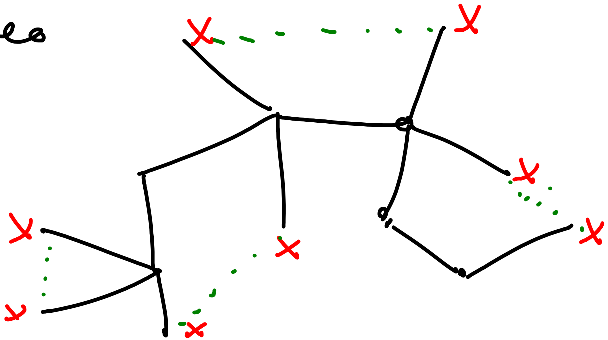
$$\ell(T) = 2 \times \ell(\text{spanning tree})$$
$$\leq 2 \times \ell(\text{minimum tour})$$

shortcut: skip visited vertices except for start

d, b, a, e, c, , d

2) $\frac{3}{2}$-approximation

(a) Find minimum spanning tree



$X = \{$ nodes of odd degree

(b) Add a matching of $X$ of minimum weight

$\leq \frac{1}{2} \ell(T^*)$

⋮