Problem: Length of a triangulation

Find
\[
\min_{\text{length of triangulation}} = \text{sum of lengths of internal edges}
\]
$M^*_{k,l} = \min \text{ length of triangulation on } X_k X_{k+1}, \ldots, X_l X_k$

$M^*_{k,l} \leq \min_{k < i < l} \left[ M^*_{k,i} + M^*_{i,l} + |X_k, X_l| \right]$

Want to compute $M^*_{j,n}$
Pig farming problem

Start of period $n$

\[ p - y \]
\[ y \]

Costs
\[ c_n(p - y) \]
Feed pig
\[ b(p - y) \]
\[ b > 1 \]

Next period

$N$ periods to go.

\[ f_n(p) = \max_{0 \leq y \leq p} \left[ R_n(y) - C_n(p - y) + f_{n+1}(b(p - y)) \right] \]
Now assume \( R_n(\infty) = R_n x \) \( \text{linear} \)

\[ C_n(\infty) = C_n x \]

Claim: \( \exists a_1, a_2, \ldots, a_{N+1} \) s.t.

\[ f_n(P) = a_n P \]

Backward Induction:

\[ f_{N+1}(P) = R_{N+1} P \]

\[ a_{N+1} = R_{N+1} \]
\[ f_n(P) = \max_{0 \leq y \leq P} \left[ R_n y - C_n (P-y) + a_{n+1} b (P-y) \right] \]

\[ = \max_{0 \leq y \leq P} \left[ a_{n+1} b P - C_n P + (R_n + C_n - a_{n+1} b) y \right] \]

\[ = \begin{cases} 
R_n P & R_n \geq a_{n+1} b - C_n \\
\left( a_{n+1} b - C_n \right) P & R_n < a_{n+1} b - C_n 
\end{cases} \]