\(8/28/09\)

\[ n = 4 \quad \alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \alpha_4 \]
\[ H = 4 \quad 5 \quad 6 \quad 4 \quad 7 \]

\[ c(x) = x(20 - x) \]

<table>
<thead>
<tr>
<th>(i)</th>
<th>(f_1(i))</th>
<th>(x_1(i))</th>
<th>(f_2(i))</th>
<th>(x_2(i))</th>
<th>(f_3(i))</th>
<th>(x_3(i))</th>
<th>(f_4(i))</th>
<th>(x_4(i))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>266</td>
<td>5</td>
<td>191</td>
<td>10</td>
<td>147</td>
<td>8</td>
<td>91</td>
<td>7</td>
</tr>
<tr>
<td>1</td>
<td>255</td>
<td>4</td>
<td>190</td>
<td>9</td>
<td>142</td>
<td>3</td>
<td>84</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>242</td>
<td>3*</td>
<td>187</td>
<td>8</td>
<td>127</td>
<td>2</td>
<td>75</td>
<td>5</td>
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<tr>
<td>3</td>
<td>227</td>
<td>2</td>
<td>182</td>
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<td>110</td>
<td>1</td>
<td>64</td>
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<tr>
<td>4</td>
<td>210</td>
<td>1</td>
<td>175</td>
<td>6</td>
<td>91</td>
<td>0*</td>
<td>51</td>
<td>3</td>
</tr>
</tbody>
</table>
Now suppose we start with 2 items in stock.

What is the optimal policy

\[ x_1 = 3 \quad x_3 = 0 \]

\[ x_2 = 10 \quad x_4 = 7 \]
Relation \( i \) shortest path problem

Starting inventory

Period \( r \)

Cost = \( c(j + d_r - i) \)
Make problem more "interesting"

(1) Add a holding cost:

\[ f_r(i) = \min \left[ c(x) + f_{i+1}(i+x-d_r) \right] \]

Suppose it costs money to store things.
Start period with inventory \( i \) if costs below:
end
\[
\underline{c(x)} \quad c(x) + h_i (i, i+x-d_r)
\]
(ii) Allow "back-ordering" up to \( B \)

Negative inventory

\[
F_i(i) = \min_x \left[ c(x) + h(i, i+x-d_r) + \gamma_{cycles}(i+n-d_r) \right]
\]

\(-B \leq i \leq H\)

\( \gamma(y) = \text{monthly cost in not filling} \)

\( y \) orders

\[
\gamma(y) = 0, \quad y \leq 0 \quad + \quad p / d_r - (i+y)
\]
(111) Numerical solutions on $\mathbb{Z}_3 \times 10, 0, 7$

An extra cost

$s(c, y) = \text{disruption cost of changing from production level } x_i \text{ to production level } y.$
\[ f(i,y) = \min_{\gamma \in \Gamma} \max_{\alpha \in \Delta} \left\{ \gamma \circ \sigma(\alpha) + \alpha \cdot (\gamma - \beta) \right\} \]

\[ \Phi(i) = \min_{\gamma \in \Gamma} \max_{\alpha \in \Delta} \left\{ \gamma \circ \sigma(\alpha) + \alpha \cdot (\gamma - \beta) \right\} \]